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Hesitant 2-tuple linguistic information in multiple attributes group decision making

Ismat Beg[a,*] and Tabasam Rashid[b]
[a]*Lahore School of Economics, Lahore, Pakistan*
[b]*Department of Mathematics, University of Management and Technology, Lahore, Pakistan*

Abstract. In this paper, the concept of a hesitant 2-tuple linguistic information model is introduced. It provides a linguistic and computational basis to manage the situations in which experts assess an alternative in linguistic term while feeling some hesitation to present its possible linguistic translations. A distance measure is defined between any two hesitant 2-tuple linguistic information. Then technique for order preference by similarity to ideal solution is formulated to solve the group decision making problem based on hesitant 2-tuple linguistic information by experts. An example is given to illustrate the practicality and feasibility of our proposed method.

Keywords: Multiple attributes group decision making, hesitant fuzzy sets, 2-tuple information

1. Introduction

Ordinary fuzzy sets have limitation for the modelling of decision problems in which two or more sources of vagueness appear. To overcome these situations different extensions of fuzzy set are given like that type-2 fuzzy sets; Nonstationary fuzzy set; Intuitionistic fuzzy set; etc. Fuzzy linguistic approach [33] has provided a useful tool in many fields and applications by experts in problems whose nature is rather qualitative [1, 24, 30]. But fuzzy linguistic approach is also not able to handle computing with words [15, 23]. So the 2-tuple fuzzy linguistic representation model was developed in [18] on the basis of the concept of symbolic translation. It can avoid the information distortion and loss in the linguistic information processing. Recently, the 2-tuple linguistic information model has been further studied and applied in the decision making problems and many aggregation operators have been developed [17, 20, 27, 28]. The 2-tuple arithmetic averaging operator, the 2-tuple arithmetic weighted averaging operator, the 2-tuple ordered weighted averaging operator and the extended 2-tuple weighted averaging operator were proposed in [19]. The extended geometric mean operator, the extended arithmetic averaging operator, the extended ordered weighted averaging operator and the extended ordered weighted geometric operator were introduced in [31]. The 2-tuple ordered weighted averaging operator and the 2-tuple ordered weighted geometric operator were studied in [22]. The extended 2-tuple ordered weighted averaging operator was proposed in [35]. The extended 2-tuple weighted geometric operator and the extended 2-tuple ordered weighted geometric operator have been defined in [29]. Herrera et al. [16] presented an unbalanced linguistic computational model that uses the 2-tuple fuzzy linguistic computational model to accomplish processes of computing with words with unbalanced term sets in a precise way and without loss of information. The concept of numerical scale and the 2-tuple fuzzy linguistic representation models was used for decision problems in [13]. The basic idea of this model is to set suitable numerical scale with the purpose of making transformations between linguistic 2-tuples and numerical values. By defining the concept of the transitive calibration matrix and its consistent index, they developed an optimization model to compute the numerical scale of

*Corresponding author. Ismat Beg, Lahore School of Economics, Lahore, Pakistan. Tel.: +92 345 4161107; Fax: +92 42 36560905; E-mail: begismat@yahoo.com.

the linguistic term set. They also constructed the transitive calibration matrix for decision problems using linguistic preference relations and analyze the linkage between the consistent index of the transitive calibration matrix and one of the linguistic preference relations. Dong et al. [10] proposed a consistency-improving model which preserves the utmost original knowledge and preferences in the process of improving consistency and it also guarantees that the elements in the optimal adjusted unbalanced linguistic preference relation are all simple unbalanced linguistic terms. Dong et al. [14] proposed an interval version of the 2-tuple fuzzy linguistic representation model. Interval multiplicative preference relations are used in the pairwise comparisons method and the interval version of the 2-tuple fuzzy linguistic representation model can be utilized in the pairwise comparisons method as it provides a novel approach to construct interval multiplicative preference relations [9]. The notion of interval valued 2-tuple fuzzy linguistic information was given by Beg and Rashid in [4]. Aggregation operators based on Choquet integral with the interval-valued 2-tuple linguistic information was also developed in [4]. Furthermore, these operators were used in multiple attribute decision making method. Recently the group decision making model based on multi-granular unbalanced 2-tuple linguistic preference relations is proposed by Dong et al. in [11]. They also gave a transformation function to relate multi-granular unbalanced linguistic preference relations with uniform balanced linguistic preference relations.

Technique for order preference by similarity to ideal solution (TOPSIS) is a useful technique for the selection of the best alternative and also for the ranking of alternatives. Hwang and Yoon [21] developed TOPSIS to multiple attribute decision making problems. TOPSIS is extended to fuzzy environment [5–7, 27]. Xu and Chen [32] used fuzzy TOPSIS for multiple attribute group decision making. Beg and Rashid [3] further extended fuzzy TOPSIS and used it for multiple attribute trapezoidal valued intuitionistic fuzzy decision making. Rodríguez et al. [25] used hesitant fuzzy linguistic term sets in decision making problems . TOPSIS is further modified for hesitant fuzzy linguistic term set to solve the multiple attribute group decision making problems in [2]. The concept of distribution assessments in a linguistic term set and the operational laws of linguistic distribution assessments were studied in [34]. The weighted averaging operator and the ordered weighted averaging operator for linguistic distribution assessments were presented and they also developed the concept of distribution linguistic preference relations, whose elements were linguistic distribution assessments. Dong et al. [12] further generalized this concept of linguistic distribution assessments with interval symbolic proportions under multi-granular unbalanced linguistic contexts; First, the weighted averaging operator and the ordered weighted averaging operator for the linguistic distribution assessments with interval symbolic proportions were presented. Then, they developed the transformation functions among the multi-granular unbalanced linguistic distribution assessments with interval symbolic proportions. Dong et al. [8] also developed an optimization-based consensus model by using consensus measure in the hesitant linguistic group decision making, which minimizes the number of adjusted simple terms in the consensus building. They displayed a two-stage model to further optimize the solutions to their proposed consensus model, through which a unique optimal adjustment suggestion is obtained to support the consensus reaching process in the hesitant linguistic group decision making.

In this paper, first we introduce the notion of hesitant 2-tuple linguistic information and then we extend fuzzy TOPSIS for hesitant 2-tuple linguistic term sets with the opinion of some decision makers about the attributes of alternatives. Next we propose a method for aggregation of the experts' opinion on different attributes for alternatives, where the opinion of the experts are represented by hesitant 2-tuple linguistic term sets. Our information model is characterized by a linguistic term and its possible symbolic translations, which is more suitable for dealing with fuzziness and uncertainty than the 2-tuple linguistic arguments. Rest of the paper is organized as follows. Some basic notions of 2-tuple linguistic information is discussed in Section 2. The concept of hesitant 2-tuple linguistic information is introduced in Section 3. The multiple attribute decision making method based on this hesitant 2-tuple linguistic information is given in Section 4. In Section 5, an example is given to illustrate the developed method and to demonstrate its practicality and feasibility. Discussion and conclusion is given in the last Section.

2. Basic concepts

The linguistic information [19] was expressed by means of 2-tuples, which were composed by a linguistic term and a numeric value assessed in $[-0.5, 0.5)$.

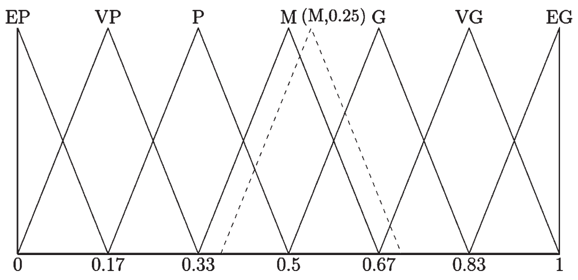

Fig. 1. 2-tuple model of (M,0.25).

Suppose that $S = \{s_i | i = 1, \dots, t\}$ is a finite and totally ordered discrete term set, where s_i represents a possible linguistic term for a linguistic variable. For example;

$S = \{s_1$ = extremely poor (EP), s_2 = very poor (VP), s_3= poor (P), s_4 = medium (M), s_5 = good (G), s_6 = very good (VG), s_7 = extremely good (EG)$\}$.

The above set satisfies the following properties:

(1) The set is ordered: $s_i \geq s_j$, if $i \geq j$;
(2) The max operator: $\max(s_i, s_j) = s_i$, if $i \geq j$;
(3) The min operator: $\min(s_i, s_j) = s_i$, if $i \leq j$.

The 2-tuple fuzzy linguistic representation model was developed based on the concept of symbolic translation. The 2-tuple (s_i, α_i) is used to represent the linguistic information, where s_i is a linguistic label from a predefined linguistic term set S and α_i is a numerical value representing the value of the possible symbolic translation and $\alpha_i \in [-0.5, 0.5)$. Suppose we have a 2-tuple model with linguistic term 'Medium (M)' and posible symbolic transalation is '0.25' then our 2-tuple model will be $(M, 0.25)$ and the structure of this model is described in Fig. 1 as a doted line. For further detail see [19].
Next we give definition of an aggregation operator ∇.

Definition 2.1. Consider k linguistic terms then the operator ∇ is defined as:

$$\nabla(s_i, s_j, ..., s_m) = s_{round\left(\frac{i+j+...+m}{k}\right)},$$

where round is the usual round operation.

3. Hesitant 2-tuple fuzzy linguistic representation model

Hesitant 2-tuple linguistic information model is introduced to manage the situations in which information described is in linguistic term and decision maker feels some hesitation to present its possible linguistic translations.

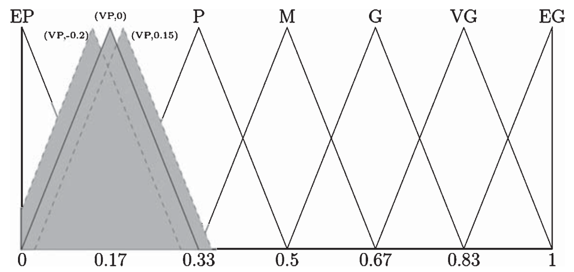

Fig. 2. Hesitant 2-tuple model (VP,(–0.2,0,0.15)).

Definition 3.1. Let X be a universe of discourse and $S = \{s_1, ..., s_t\}$ be a linguistic term set, a hesitant linguistic term set in X is an expression A given by $A = \{(x, h(x)) | x \in X\}$, where $h(x) = (s_i, \beta_{ij})$ for all x in X.

The hesitant 2-tuple fuzzy linguistic representation model represents the hesitant linguistic information by means of a 2-tuple, (s_i, β_{ij}), where s_i is linguistic label and β_{ij} is a finite subset of $[-0.5, 0.5)$ that represent the possible symbolic translations of s_i. It is noted that the cardinality of β may be different for each x.

Example 3.2. To evaluate the happiness index of three family members (Joe, Aron, Alam) in a family for the predefined linguistic term set S. The people may think that their happiness index is VP, M and G, respectively. They may hesitate to give the translation values to these linguistic terms. They are double minded/confuse in these translation values $(-0.2, 0, 0.15)$, $(-0.4, -0.3)$ and $(-0.1, 0.2, 0.32)$ for Joe, Aron and Alam, respectively. So the hesitant 2-tuple model of the happiness index will $(VP, (-0.2, 0, 0.15))$, $(M, (-0.4, -0.3))$ and $(G, (-0.1, 0.2, 0.32))$ for Joe, Aron and Alam, respectively. In the setting of hesitant 2-tuple linguistic term set, it can be written as: $\{(Joe, (VP, (-0.2, 0, 0.15)))$, $(Aron, (M, (-0.4, -0.3)))$, $(Alam, (G, (-0.1, 0.2, 0.32)))\}$.

The hesitant 2-tuple model of Joe happiness $(Joe, (VP, (-0.2, 0, 0.15)))$ is shown in Fig. 2.
Distance measure is important to solve many decision making problems. Here we propose a formula to calculate the distance between any two hesitant 2-tuple linguistic arguments.

Definition 3.3. Let (s_i, β_{ij}) and (s_l, β_{lk}) be two hesitant 2-tuple linguistic arguments, such that $\beta_{ij} = \{a_1, a_2, \dots, a_n\}$ and $\beta_{lk} = \{b_1, b_2, \dots, b_m\}$, then distance '$d$' between (s_i, β_{ij}) and (s_l, β_{lk}) is defined as follows:

$$d((s_i, \beta_{ij}), (s_l, \beta_{lk})) = |i-l| + \max\left\{\begin{array}{l} \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{b_i \in \beta_{lk}} (|a_i - b_i|)\right\}, \\ \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{a_i \in \beta_{ij}} (|a_i - b_i|)\right\} \end{array}\right\}$$

Proposition 3.4. *Let (s_i, β_{ij}), (s_l, β_{lk}) and (s_f, β_{fv}) be hesitant 2-tuple linguistic arguments, such that $\beta_{ij} = \{a_1, a_2, \ldots, a_n\}$, $\beta_{lk} = \{b_1, b_2, \ldots, b_m\}$ and $\beta_{fv} = \{c_1, c_2, \ldots, c_w\}$, then distance '$d$' between (s_i, β_{ij}) and (s_l, β_{lk}) satisfies the following properties.*

1. $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) \geq 0$
2. $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) = 0$ *if and only if* $(s_i, \beta_{ij}) = (s_l, \beta_{lk})$
3. $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) = d((s_l, \beta_{lk}), (s_i, \beta_{ij}))$
4. $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) \leq d((s_l, \beta_{lk}), (s_f, \beta_{fv})) + d((s_f, \beta_{fv}), (s_i, \beta_{ij}))$.

Proof.

1. As we know that $|y| \geq 0$ for any $y \in \Re$, thus $|a_i - b_i| \geq 0$ and $|i-l| \geq 0$.

   Since the sum of positive numbers is also a positive number. Therefore $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) \geq 0$.
2. $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) = 0 \Leftrightarrow$

   $$|i-l| + \max\left\{\begin{array}{l} \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{b_i \in \beta_{lk}} (|a_i - b_i|)\right\}, \\ \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{a_i \in \beta_{ij}} (|a_i - b_i|)\right\} \end{array}\right\} = 0$$

   $\Leftrightarrow |i-l| = 0$ and $|a_i - b_i| = 0$ for all a_i and b_i.

   $\Leftrightarrow (s_i, \beta_{ij}) = (s_l, \beta_{lk})$.
3. Since $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) = |i-l| +$

   $$\max\left\{\begin{array}{l} \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{b_i \in \beta_{lk}} (|a_i - b_i|)\right\}, \\ \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{a_i \in \beta_{ij}} (|a_i - b_i|)\right\} \end{array}\right\}$$

   Also $|y-z| = |z-y|$ for any $z, y \in \Re$, therefore $|i-l| = |l-i|$ and $|a_i - b_i| = |b_i - a_i|$.

   Thus, it can be written as $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) =$

   $$|l-i| + \max\left\{\begin{array}{l} \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{a_i \in \beta_{ij}} (|b_i - a_i|)\right\}, \\ \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{b_i \in \beta_{lk}} (|b_i - a_i|)\right\} \end{array}\right\} =$$

   $d((s_l, \beta_{lk}), (s_i, \beta_{ij}))$.
4. Since $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) = |i-l| +$

   $$\max\left\{\begin{array}{l} \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{b_i \in \beta_{lk}} (|a_i - b_i|)\right\}, \\ \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{a_i \in \beta_{ij}} (|a_i - b_i|)\right\} \end{array}\right\}$$

   Also $|y-z| \leq |y-u| + |u-z|$ for any $y, z, u \in \Re$. So $|i-l| \leq |i-f| + |f-l|$ and $|a_i - b_i| \leq |a_i - c_i| + |a_i - c_i|$.

   Therefore,

   $$d((s_i, \beta_{ij}), (s_l, \beta_{lk})) \leq |i-f| + |f-l| + \max\left\{\begin{array}{l} \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{c_i \in \beta_{fv}} (|a_i - c_i|)\right\}, \\ \max\limits_{c_i \in \beta_{fv}} \left\{\min\limits_{a_i \in \beta_{ij}} (|a_i - c_i|)\right\} \end{array}\right\} + \max\left\{\begin{array}{l} \max\limits_{c_i \in \beta_{fv}} \left\{\min\limits_{b_i \in \beta_{lk}} (|c_i - b_i|)\right\}, \\ \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{c_i \in \beta_{fv}} (|c_i - b_i|)\right\} \end{array}\right\}$$

   $$\leq |i-f| + \max\left\{\begin{array}{l} \max\limits_{a_i \in \beta_{ij}} \left\{\min\limits_{c_i \in \beta_{fv}} (|a_i - c_i|)\right\}, \\ \max\limits_{c_i \in \beta_{fv}} \left\{\min\limits_{a_i \in \beta_{ij}} (|a_i - c_i|)\right\} \end{array}\right\} + |f-l| + \max\left\{\begin{array}{l} \max\limits_{c_i \in \beta_{fv}} \left\{\min\limits_{b_i \in \beta_{lk}} (|c_i - b_i|)\right\}, \\ \max\limits_{b_i \in \beta_{lk}} \left\{\min\limits_{c_i \in \beta_{fv}} (|c_i - b_i|)\right\} \end{array}\right\}.$$

   It yields that $d((s_i, \beta_{ij}), (s_l, \beta_{lk})) \leq d((s_l, \beta_{lk}), (s_f, \beta_{fv})) + d((s_f, \beta_{fv}), (s_i, \beta_{ij}))$. ■

Example 3.5. Let $(EG, (0.1, 0.2, 0.4))$ and $(M, (-0.3, -0.1))$ be two hesitant 2-tuple linguistic arguments. Then distance between these arguments is calculated below:
$d((EG, (0.1, 0.2, 0.4)), (M, (-0.3, -0.1)))$
$= 3 + 0.5 = 3.5$.

Definition 3.6. Consider finite hesitant 2-tuple linguistic arguments are (s_k, β_{kc}), (s_i, β_{ij}), ..., (s_l, β_{lv}), further suppose that $\max(s_k, s_i, ..., s_l) = s_i$, then

$\max((s_k, \beta_{kc}), (s_i, \beta_{ij}), ..., (s_l, \beta_{lv})) = (s_i, (x | x \in \beta_{ij}$ and $\max(\beta_{ij}) \leq x \leq \max(\beta_{iu})$ for all $j \neq u$ and specific i)).

Example 3.7. Let (EG,(–0.4,–0.3,0.1), (VG,(0.2,–0.1, 0)) and (EG,(0.2,0.4)) be three hesitant 2-tuple linguistic arguments, then

$\max((EG, (-0.4, -0.3, 0.1), (VG, (0.2, -0.1, 0)), (EG, (0.2, 0.4))) = (EG, (0.1, 0.2, 0.4))$.

4. TOPSIS to multiple attributes group decision making

In general, multiple attributes group decision making problem includes uncertain and imprecise data and information. We consider the multiple attributes group decision making problems where all the attributes' values are expressed in hesitant 2-tuple linguistic information. In group decision making problems, aggregation of expert opinions is very important to perform evaluation process. In this section, TOPSIS is proposed for multiple attributes hesitant 2-tuple linguistic group decision making. The TOPSIS is based on the following steps:

Step 1. Let $D = \{D_1, D_2, \ldots, D_K\}$ be the set of decision makers, $A = \{A_1, A_2, \ldots, A_m\}$ be the set of alternatives and $B = \{B_1, B_2, \ldots, B_n\}$ be the set of attributes.

Step 2. The decision maker D_l evaluates the alternative A_i with respect to the attribute B_j to get $\gamma_{ij}^{(l)}$, then the decision matrices $R^{(l)} = \left[\left(s_{ij}^{(l)}, x_{ij}^{(l)}\right)\right]_{m\times n}$ $(l = 1, 2, \ldots, K)$ are formed.

Step 3. We calculate the one decision matrix X by aggregating the opinions of decision makers $(\tilde{X}^1, \tilde{X}^2, \cdots, \tilde{X}^K)$; $X = [(s_{ij}, x_{ij})]$, such that $s_{ij} = \nabla(s_{ij}^1, s_{ij}^2, ..., s_{ij}^K)$ and $x_{ij} = \{x \mid x \in x_{ij}^{(l)}$ and $r_{p_{ij}} \leq x \leq r_{q_{ij}}$ for all $l\}$ where

$$r_{p_{ij}} = \min\left\{\min_{l=1}^{K}(\max x_{ij}^{(l)}), \max_{l=1}^{K}(\min x_{ij}^{(l)})\right\}$$

and

$$r_{q_{ij}} = \max\left\{\min_{l=1}^{K}(\max x_{ij}^{(l)}), \max_{l=1}^{K}(\min x_{ij}^{(l)})\right\}.$$

Performance of alternative A_i with respect to attribute C_j is denoted as (s_{ij}, x_{ij}), in an aggregated matrix X.

Step 4. Let Ω_b be the collection of benefit attributes (i.e., the larger C_j, the greater preference) and Ω_c be the collection of cost attributes (i.e., the smaller C_j, the greater preference). The hesitant 2-tuple linguistic positive-ideal solution, denoted by $\tilde{A}^+ = (\tilde{V}_1^+ \ \tilde{V}_2^+ \ \ldots \ \tilde{V}_n^+)$, and the hesitant 2-tuple linguistic negative-ideal solution, denoted by $\tilde{A}^- = (\tilde{V}_1^- \ \tilde{V}_2^- \ \ldots \ \tilde{V}_n^-)$, are defined as follows: $\tilde{A}^+ = \left[\max_{l=1}^{K}\left(\max_i\left(s_{ij}^{(l)}, x_{ij}^{(l)}\right)\right) \forall i|\, j \in \Omega_b, \min_{l=1}^{K}\left(\min_i\left(s_{ij}^{(l)}, x_{ij}^{(l)}\right)\right) \forall i|\, j \in \Omega_c\right]$ $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$\tilde{A}^+ = (\tilde{V}_1^+ \ \tilde{V}_2^+ \ \ldots \ \tilde{V}_n^+)$$

$\tilde{A}^- = \left[\min_{l=1}^{K}\left(\min_i\left(s_{ij}^{(l)}, x_{ij}^{(l)}\right)\right) \ \forall \ i|j \in \Omega_b, \max_{l=1}^{K}\left(\max_i\left(s_{ij}^{(l)}, x_{ij}^{(l)}\right)\right) \forall i|j \in \Omega_c\right]$ $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

$$\tilde{A}^- = (\tilde{V}_1^- \ \tilde{V}_2^- \ \ldots \ \tilde{V}_n^-)$$

Step 5. Construct positive ideal separation matrix (D^+) and negative ideal separation matrix (D^-) which are defined as follows:

$$D^+ = \begin{bmatrix} d(x_{11}, \tilde{V}_1^+) & + & \cdots & + & d(x_{1n}, \tilde{V}_n^+) \\ d(x_{21}, \tilde{V}_1^+) & + & \cdots & + & d(x_{2n}, \tilde{V}_n^+) \\ \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}_1^+) & + & \cdots & + & d(x_{mn}, \tilde{V}_n^+) \end{bmatrix} = \begin{bmatrix} d(A_1, \tilde{A}^+) \\ d(A_2, \tilde{A}^+) \\ \vdots \\ d(A_m, \tilde{A}^+) \end{bmatrix}$$

and

$$D^- = \begin{bmatrix} d(x_{11}, \tilde{V}_1^-) & + & \cdots & + & d(x_{1n}, \tilde{V}_n^-) \\ d(x_{21}, \tilde{V}_1^-) & + & \cdots & + & d(x_{2n}, \tilde{V}_n^-) \\ \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}_1^-) & + & \cdots & + & d(x_{mn}, \tilde{V}_n^-) \end{bmatrix} = \begin{bmatrix} d(A_1, \tilde{A}^-) \\ d(A_2, \tilde{A}^-) \\ \vdots \\ d(A_m, \tilde{A}^-) \end{bmatrix}$$

Step 6. Calculate the relative closeness (RC) of each alternative to the ideal solution as follows:

$$RC(A_i) = \frac{d(A_i, A^-)}{d(A_i, A^+) + d(A_i, A^-)}, \quad i = 1, 2, \ldots, m,$$

Step 7. Rank all the alternatives A_i $(i = 1, 2, \ldots, m)$ according to the closeness coefficient $RC(A_i)$, the greater the value $RC(A_i)$, the better the alternative A_i.

5. Example

Assume that there is a finance house, who wants to invest money in the best option.

Step 1. There are five possible alternatives in which to invest the money: A_1 is a refrigerator company; A_2 is a food company; A_3 is a construction company; A_4 is movies industry; A_5 is a software house. Suppose that there are three decision makers/directors D_l $(i = 1, 2, 3)$ in the committee and four attributes B_i $(i = 1, 2, 3, 4)$ are used to evaluate the alternatives: B_1–growth factor; B_2–tax problems; B_3–risk issue; B_4–social impact.

Step 2. The decision makers evaluate the alternatives with respect to the attributes in hesitant 2-tuple linguistic arguments to form decision matrices $\tilde{R}^{(l)} = \left(r_{ij}^{(l)}\right)_{5\times 4}$ $(l = 1, 2, 3)$, as shown in Tables 1–3.

Step 3. Aggregate all the decision matrices $\tilde{R}^{(i)}$ $(i = 1, 2, 3)$, into the collective decision matrix $\tilde{R}_{5\times 4}$ as in Table 4.

Step 4. For cost attributes B_2, B_3 and benefit attributes B_1, B_4, the positive and negative ideal solutions are in Tables 5 and 6, respectively.

Step 5. Positive ideal separation matrix:

$$D^+ = \begin{bmatrix} 3+0.5 + 3+0.1 + 1+ 0.65 + 4+0.15 \\ 4+0.2 + 0+0.2 + 3+0.5 + 2+0.45 \\ 3+0.3 + 1+0.15 + 3+ 0.55 + 5+0.55 \\ 1+0.2 + 1+0.2 + 1+0.4 + 2+0.5 \\ 2+0.3 + 0+0.2 + 1+0.3 + 1+0.3 \end{bmatrix} = \begin{bmatrix} 12.4 \\ 10.35 \\ 13.55 \\ 6.3 \\ 5.1 \end{bmatrix}$$

Negative ideal separation matrix:

$$D^- = \begin{bmatrix} 2+0.7 + 0+0.2 + 3+0.4 + 1+0.4 \\ 1+0.5 + 3+0.1 + 1+0.1 + 3+0.1 \\ 2+0.3 + 2+0.15 + 1+0.3 + 0+0 \\ 4+0.4 + 2+0.1 + 3+0.2 + 2+0.1 \\ 3+0.5 + 3+0.3 + 3+0.3 + 4+0.25 \end{bmatrix} = \begin{bmatrix} 7.7 \\ 8.8 \\ 5.75 \\ 11.8 \\ 14.35 \end{bmatrix}$$

Step 6. Relative closeness of the alternatives: $RC(A_1) = 0.38308$, $RC(A_2) = 0.45953$, $RC(A_3) = 0.29792$, $RC(A_4) = 0.65193$ and $RC(A_5) = 0.7377$.

Step 7. Ranking of the alternatives:

$$A_3 \prec A_1 \prec A_2 \prec A_4 \prec A_5.$$

Therefore, the most desirable alternative to invest the money is software house.

6. Discussion and conclusion

We describe briefly several 2-tuple models and contrast them with the proposal presented in this paper:

Herrera and Martínez [19] proposed a symbolic model called the 2-tuple linguistic representation model, to perform the process of computing with words without loss of any information. In their proposed 2-tuple model, the linguistic term sets were uniformly and symmetrically distributed. Based on the Herrera and Martínez model, the following three different models have been studied:

- Wang and Hao model [27],
- Herrera et al. model [16],
- Numerical scale model [9, 10, 13, 14].

In all these models linguistic term sets have been studied where they were uniformly and symmetrically distributed. Meanwhile, Herrera and Martínez [20] investigated the multi-granular linguistic decision-making based on the 2-tuple linguistic model. However, the linguistic terms used in this model was single linguistic term.

The linguistic distribution assessment in [34] is a natural generation of the proportional 2-tuples proposed in the Wang and Hao model [27]. However, the linguistic distribution assessment presented in Zhang et al. [34] is based on the Herrera and Martínez model [19], and the linguistic term sets used are uniformly and symmetrically distributed. In addition, the symbolic proportions over linguistic terms are exact values, and only one linguistic term set is considered. Torra [26] introduced the concept of hesitant fuzzy set as an extension of ordinary fuzzy set to handle the situations in which we have a set of possible membership values instead of single membership degree. The motivation is that when defining the linguistic term in a 2-tuple model, the difficulty of establishing the possible translation value of this linguistic term on the possible values, but because we have a set of possible values. This is the case if we consider as possible values for the translation of linguistic term in the 2-tuple model. This situation can arise in a multi criteria decision making problem. According to the above comparison, the proposal in this paper incorporates many practical new decision situations.

Table 1
The decision matrix $\tilde{R}^{(1)}$

	B_1	B_2	B_3	B_4
A_1	$(M, (-0.3, 0, 0.2))$	$(G, (0.45, 0.32, 0.2))$	$(P, (0.2, 0.3))$	$(P, (-0.3, 0.1))$
A_2	$(P, (0, 0.2, 0.1))$	$(M, (-0.48, -0.2, 0))$	$(M, (-0.45, 0.1))$	$(G, (-0.2, 0.1, 0.2))$
A_3	$(G, (0.1, -0.3, 0.2))$	$(M, (-0.1, 0.2))$	$(VG, (-0.2, 0, 0.4))$	$(P, (-0.3, 0.1, 0.2))$
A_4	$(VG, (0.2, -0.1, 0))$	$(P, (0, 0.2, 0.4))$	$(P, (-0.5, -0.3))$	$(M, (-0.45, -0.25))$
A_5	$(EG, (-0.4, -0.3, 0.1))$	$(P, (-0.1, 0.2, 0.3))$	$(VP, (-0.45, -0.2))$	$(G, (-0.4, -0.1, 0))$

Table 2
The decision matrix $\tilde{R}^{(2)}$

	B_1	B_2	B_3	B_4
A_1	$(P, (-0.3, -0.1))$	$(VG, (-0.1, 0, 0.1))$	$(VP, (-0.2, 0.3))$	$(M, (0.1, 0.2, 0.4))$
A_2	$(VP, (0.4))$	$(P, (0.2, 0.3))$	$(G, (0.3, 0.4))$	$(VG, (-0.1, -0.45, -0.2))$
A_3	$(M, (0.1, 0.3))$	$(P, (0.2, -0.1))$	$(G, (0.1, 0.3))$	$(VP, (-0.2, -0.3, 0))$
A_4	$(EG, (0.2, 0.4))$	$(M, (-0.4, 0.3))$	$(P, (0.2, 0.4))$	$(G, (0.1, 0.3, 0.4))$
A_5	$(G, (-0.2, 0.1))$	$(M, (-0.2, 0.15))$	$(P, (-0.1, 0.2))$	$(VG, (-0.1, 0.3))$

Table 3
The decision matrix $\tilde{R}^{(3)}$

	B_1	B_2	B_3	B_4
A_1	$(G, (-0.5, 0.1, 0.2))$	$(VG, (0.2, 0.3))$	$(M, (0.1, 0.2))$	$(VP, (0, 0.1, 0.2))$
A_2	$(M, (-0.4, -0.1))$	$(P, (0, 0.2, 0.4))$	$(VG, (-0.2, -0.3))$	$(M, (-0.1, -0.2, 0))$
A_3	$(P, (-0.2, 0, 0.1))$	$(VG, (-0.05, 0.2))$	$(G, (0, 0.1, 0.25))$	$(VP, (-0.2, -0.3, 0))$
A_4	$(G, (-0.3, -0.1, 0))$	$(G, (0, 0.25, 0.45))$	$(P, (0.2, 0.3, 0.1))$	$(M, (-0.1, 0.2, 0.3))$
A_5	$(M, (-0.1, 0.1, 0.3))$	$(P, (-0.1, -0.2, 0))$	$(M, (0.1, 0.4, 0.45))$	$(EG, (-0.05, 0.25))$

Table 4
The decision matrix $\tilde{R}$

	B_1	B_2	B_3	B_4
A_1	$(M, (-0.3, -0.1))$	$(VG, (-0.1, 0.1, 0.2))$	$(P, (0.2))$	$(P, (0.1))$
A_2	$(P, (-0.1, 0, 0.1, 0.2, 0.4))$	$(P, (0, 0.2))$	$(G, (-0.2, 0.1, 0.3))$	$(G, (-0.2, -0.1))$
A_3	$(M, (0.1))$	$(M, (-0.05, 0.2))$	$(G, (0.1, 0.25))$	$(VP, (-0.3, -0.2, 0))$
A_4	$(VG, (0, 0.2))$	$(M, (0, 0.2, 0.25, 0.3))$	$(P, (-0.3, 0.1, 0.2))$	$(M, (-0.25, -0.1, 0.1))$
A_5	$(G, (-0.1, 0.1))$	$(P, (-0.1, 0))$	$(P, (-0.2, -0.1, 0.1))$	$(VG, (-0.05, 0))$

Table 5
The positive ideal solution

	B_1	B_2	B_3	B_4
$\tilde{A}^+$	$(EG, (0.1, 0.2, 0.4))$	$(P, (-0.1, -0.2, 0, 0.2))$	$(VP, (-0.45, -0.2,))$	$(EG, (0.25))$

Table 6
The negative ideal solution

	B_1	B_2	B_3	B_4
$\tilde{A}^-$	$(VP, (0.4))$	$(VG, (0.1, 0.2, 0.3))$	$(VG, (-0.2, 0, 0.4))$	$(VP, (-0.3, -0.2, 0))$

We studied the situation where the attributes in the decision making problem are evaluated by hesitant 2-tuple linguistic arguments. Aggregation procedure is defined for hesitant 2-tuple linguistic information and this procedure is applied to the multiple attributes decision making problem. Finally, an example has been constructed to show the proposed group decision making method. The proposed method is different from all the previous techniques for group decision making due to the fact that the proposed method use hesitant 2-tuple fuzzy linguistic information, which will not cause any loss of information in the process. So it is efficient and feasible for real-world decision making applications.

References

[1] H. Becker, Computing with words and machine learning in medical diagnosis, *Information Sciences* **134** (2001), 53–69.

[2] I. Beg and T. Rashid, TOPSIS for hesitant fuzzy linguistic term sets, *International Journal of Intelligent Systems* **28**(12) (2013), 1162–1171.

[3] I. Beg and T. Rashid, Multi criteria trapezoidal valued intuitionistic fuzzy decision making with Choquet integral based TOPSIS, *OPSEARCH* **51**(1) (2014), 98–129.

[4] I. Beg and T. Rashid, Aggregation operators of interval valued 2-tuple linguistic information, *International Journal of Intelligent Systems* **29** (2014), 634–667.

[5] C.T. Chen, Extensions of the TOPSIS for group decisionmaking under fuzzy environment, *Fuzzy Sets and Systems* **114** (2000), 1–9.

[6] S.J. Chen and C.L. Hwang, *Fuzzy multiple attribute decision making*. Berlin: Springer, 1992.

[7] T.C. Chu and Y.C. Lin, An interval arithmetic based fuzzy TOPSIS model, *Expert Systems with Applications* **36** (2009), 10870–10876.

[8] Y.C. Dong, X. Chen and F. Herrera, Minimizing adjusted simple terms in the consensus reaching process with hesitant linguistic assessments in group decision making, *Information Sciences* **297** (2015), 95–117.

[9] Y.C. Dong and E. Herrera-Viedma, Consistency-driven automatic methodology to set interval numerical scales of 2-tuple linguistic term sets and its use in the linguistic GDM with preference relations, *IEEE Transactions on Cybernetics* **45**(4) (2015), in press, DOI: 10.1109/TCYB.2014.2336808

[10] Y.C. Dong, C.C. Li and F. Herrera, An optimization-based approach to adjusting the unbalanced linguistic preference relations to obtain a required consistency level, *Information Sciences* **292** (2015), 27–38.

[11] Y.C. Dong, C.C. Li, Y.F. Xu and X. Gu, Consensus-based group decision making under multi-granular unbalanced 2-tuple linguistic preference relations, *Group Decision and Negotiation* **24** (2015), 217–242.

[12] Y.C. Dong, Y.Z. Wu, H.J. Zhang and G.Q. Zhang, Multigranular unbalanced linguistic distribution assessments with interval symbolic proportions, *Knowledge-Based Systems* **82**(C) (2015), 139–151.

[13] Y.C. Dong, Y.F. Xu and S. Yu, Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model, *IEEE Transactions on Fuzzy Systems* **17** (2009), 1366–1378.

[14] Y.C. Dong, G.Q. Zhang, W.C. Hong and S. Yu, Linguistic computational model based on 2-tuples and intervals, *IEEE Transactions on Fuzzy Systems* **21** (2013), 1006–1018.

[15] F. Herrera, S. Alonso, F. Chiclana and E. Herrera-Viedma, Computing with words in decision making: Foundations, trends and prospects, *Fuzzy Optimization and Decision Making* **8**(4) (2009), 337–364.

[16] F. Herrera, E. Herrera-Viedma and L. Martínez, A fuzzy linguistic methodology to deal with unbalanced linguistic term sets, *IEEE Transactions on Fuzzy Systems* **16**(2) (2008), 354–370.

[17] F. Herrera, E. Herrera-Viedma and L. Martinez, A fusion approach for managing multi-granularity linguistic term sets in decision making, *Fuzzy Sets and Systems* **114** (2000), 43–58.

[18] F. Herrera and L. Martinez, An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making, *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems* **8**(5) (2000), 539–562.

[19] F. Herrera and L. Martinez, A 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Transactions on Fuzzy Systems* **8** (2000), 746–752.

[20] F. Herrera and L. Martinez, A model based on linguistic 2-tuple for dealing with multi-granular hierarchical linguistic contexts in multi-expert decision-making, *IEEE Transactions on Systems, Man, and Cybernetics* **31** (2001), 227–234.

[21] C.L. Hwang and K. Yoon, Multiple attributes decision making methods and applications, Berlin, Heidelberg: Springer, 1981.

[22] Y.P. Jiang and Z.P. Fan, Property analysis of the aggregation operators for 2-tuple linguistic information, *Control and Decision* **18**(6) (2003), 754–757.

[23] J. Kacprzyk and S. Zadrozny, Computing with words is an implementable paradigm: Fuzzy queries, linguistic data summaries, and natural-language generation, *IEEE Transactions on Fuzzy Systems* **18**(3) (2010), 461–472.

[24] L. Martínez, Sensory evaluation based on linguistic decision analysis, *International Journal of Approximate Reasoning* **44**(2) (2007), 148–164.

[25] R.M. Rodríguez, L. Martínez and F. Herrera, Hesitant fuzzy linguistic term sets for decision making, *IEEE Transactions on Fuzzy Systems* **20**(1) (2012), 109–119.

[26] V. Torra, Hesitant fuzzy sets, *International Journal of Intelligent Systems* **25** (2010), 529–539.

[27] J.H. Wang and J.Y. Hao, A new version of 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Transactions on Fuzzy Systems* **14**(3) (2006), 435–445.

[28] G.W. Wei, Extension of TOPSIS method for 2-tuple linguistic multiple attribute group decision making with incomplete weight information, *Knowledge and Information Systems* **25** (2010), 623–634.

[29] G.W. Wei, Method for two-tuple linguistic group decision-making based on the ET-WG and ET-OWG operators, *Expert Systems with Applications* **37** (2010), 7895–7900.

[30] D. Wu and J.M. Mendel, Computing with words for hierarchical decision making applied to evaluating a weapon system, *IEEE Transactions on Fuzzy Systems* **18**(3) (2010), 441–460.

[31] Z.S. Xu, EOWA and EOWG operators for aggregating linguistic labels based on linguistic preference relations, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **12** (2004), 791–810.

[32] Z.S. Xu and J. Chen, An interactive method for fuzzy multiple attribute group decision making, *Information Sciences* **177**(1) (2007), 248–263.

[33] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, *Information Sciences* **8**(3) (1975), 199–249.

[34] G.Q. Zhang, Y.C. Dong and Y.F. Xu, Consistency and consensus measures for linguistic preference relations based on distribution assessments, *Information Fusion* **17** (2014), 46–55.

[35] Y. Zhang and Z.P. Fan, An approach to linguistic multiple attribute decision-making with linguistic information based on ELOWA operator, *Systems Engineer* **24**(12) (2006), 324–339.