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Heat and mass transfer on the peristaltic transport of non-Newtonian fluid with creeping flow

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ABSTRACT

A new mathematical model has been developed for the peristaltic transport of Maxwell fluid with heat and mass transfer, while taking into account the effect of thermal diffusion (Soret), occurring in an asymmetric channel with creeping flow. The inertia terms are omitted from the equations of motion, which leads to solutions that approximately valid for low Reynolds number, i.e. $Re \ll 1$. The walls are kept at different but constant temperatures and concentrations. A perturbation solution is acquired, which satisfies the momentum, energy and concentration equations for the case by choosing a small wave number. Numerical results are evaluated for pressure rise and frictional forces per wavelength. The velocity, temperature and concentration fields have been appraised for diverse values of the parameters entering into the problem. The influence of diverse parameters of interest on pumping, trapping, temperature and concentration profiles has been investigated graphically.

1. Introduction

Peristaltic pumping is a form of physiological fluid transport that occurs in the human body. Peristaltic action is an inherent neromuscular property of any tubular smooth muscle structure. The study of heat and mass transfer on the peristaltic transport has several industrial applications. Typical applications include processes such as drying, evaporation at the surface of a water body, energy transfer in wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. When heat and mass transfer occur simultaneously in a moving fluid, the relationship between the fluxes and the driving potentials is of more intricate nature. Mass fluxes can be created by temperature gradient which is also known as the Soret or thermal-diffusion effect. The Soret effect has been used for isotope separation and in mixture between gases with very light molecular weight (H_2,He) and of medium molecular weight (H_2,air) Gupta and Gupta [1]. In many previous studies the Soret effect has been neglected, on the basis that it is of a smaller order of magnitude than the effects described by Fick's laws: Radhakrishnamacharya and Radhakrishnamacharya [2], Chamkha and Khaled [3], Khan et al. [4], Postelnicu [5], Mekheimer [6] and Ogulu [7].

Mathematical models have already been derived for a train of periodic sinusoidal wave in an infinite or finite two dimensional symmetric or axi-symmetric channel/tubes containing Newtonian or non-Newtonian fluids. In this direction first initiative was taken by Latham [8]. Burns and Parkes [9] have investigated the peristaltic flow through axially symmetrical pipes and channel under the effect of creeping flow. Barton and Raynor [10] have investigated the peristaltic flow in tubes with approximation of low Reynolds number. Shapiro et al. [11] has also used the long wave length approximation in studying the peristaltic pumping phenomenon. Jaffrin and Shapiro [12] have investigated peristaltic pumping with neglecting inertia terms. Gupta and Seshadri [13] have analyzed the peristaltic flow through non-uniform channels and tubes with reference to the flow of spermatic fluid in vas deferens, neglecting the inertia terms. Srivastava and Srivastava [14] have extended the analysis of peristaltic transport to two layered model. Mekheimer [6] has studied the peristaltic transport of a viscous fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non uniform and has a sinusoidal wave traveling down its wall and the inner one is rigid, uniform tube and moving with a constant velocity.

Further, several authors have attempted to solve momentum equation related to peristaltic flows with diverse approximations along with associated heat and mass transfer. However, the creeping flows (low Reynolds number) have not been considered in the aforementioned work.

Major recent research related to the peristaltic transport with heat and mass transfer for Newtonian and non-Newtonian fluid includes the following works. Radhakrishnamacharya and Srinivasulu [15] have examined the influence of wall properties on peristaltic transport with heat transfer. Srinivas and Kothandapani [16] have done peristaltic transport in an asymmetric channel with

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heat transfer. Another approach consists of the studying the effect of heat transfer and magnetic induction on the viscous fluid in a vertical annulus Mekheimer and Elmabound [17]. Eldabe et al. [18] has studied the problem of peristaltic transport of a non-Newtonian fluid with variable viscosity in the presence of heat and mass transfer along with mixed diffusion flow between a vertical wall that deforms the shape of a travelling wave and a parallel flat wall. Srinivas and Kothandapani [19] have investigated the effects of heat and mass transfer on peristaltic transport in a porous space with compliant walls. Srinivas and Gayathri [20] have reported the peristaltic flow of viscous fluid in an asymmetric channel with heat transfer and porous medium.

Nadeem and Akbar [21] has discussed the influence of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer. Srinivas et al. [22] have investigated mixed convective heat and mass transfer in an asymmetric channel with peristalsis. El-Sayed et al. [23] has studied the effect of mass diffusion of chemical species on peristaltic transport through the vertical porous media in the gap between concentric tubes with heat and mass transfer. El-Sayed et al. [24] has also been studied magnetothermodynamic peristaltic flow of Bingham non-Newtonian fluid in eccentric annuli with slip velocity and temperature jump conditions. Tripathi and Beg [25] have studied the unsteady physiological magneto-fluid flow and heat transfer through a finite length channel by peristaltic pumping.

Through an extensive survey of existing literature one can say that creeping effect for non-Newtonian fluids for the peristaltic transport have not come under considerable attention. In contrast to existing work, this paper considers heat and mass transfer on peristaltic transport of creeping flow. The objective of the present work is to extend the flow analysis of peristaltic mechanism of Maxwell fluid in an asymmetric channel as presented in Hayat et al. [26] to creeping flow with Soret effect. The novelty of this work is that after applying the creeping flow assumption to Maxwell fluid, the effect of non-Newtonian fluid are incorporated. In this investigation, a mathematical model is presented to understand the influence of creeping flow on the peristaltic transport of Maxwell fluid in an asymmetric channel with heat and mass transfer. The momentum, energy and concentration equations are simplified by neglecting inertia terms and analytic solutions for the flow variables have been derived. The features of the flow characteristics are analyzed by plotting graphs and discussed in details. The contributions of Soret number in particular, and those of the geometrical parameters in general, to the flow, heat and mass transfer characteristics are found to be quite significant and interesting.

The rest of this paper has been organized as follows: Mathematical model has been presented in Section 2, problem has been formulated in Section 3, Perturbation based solution has been presented in Section 4 and detailed discussion about the results has been given in Section 5. Finally, some conclusions have been drawn out in Section 6.

2. Mathematical model

The motion of the steady, heat and mass transfer equations of Maxwell fluid is governed by the following system of equations:

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\rho(\mathbf{V}.\nabla)\mathbf{V} = \nabla\mathbf{T}' + \rho\mathbf{b} + \mathbf{R}, \tag{2}$$

$$\mathbf{T}'.\mathbf{L} - \nabla.\mathbf{q} + \rho\mathbf{r} = \rho(\mathbf{V}.\nabla)e, \tag{3}$$

$$(\mathbf{V}.\nabla)C = D.\nabla(\nabla C) + \frac{DK_T}{T_m}\nabla.(\nabla.\mathbf{T}) - k_1C, \tag{4}$$

where

$$\mathbf{T}' = -p\mathbf{I} + \mathbf{S}, \tag{5}$$

$$\mathbf{S} + \Lambda_1\left(\frac{d\mathbf{S}}{dt} - \mathbf{LS} - \mathbf{SL}\right) = \mu\mathbf{A}_1, \tag{6}$$

$$\mathbf{L} = \nabla.\mathbf{V}, \tag{7}$$

$$\mathbf{A}_1 = (\nabla.\mathbf{V}) + (\nabla.\mathbf{V})^t. \tag{8}$$

Here, $\mathbf{V}$ is the velocity vector, $\mathbf{b}$ is the body force (assumed to be zero), $\mathbf{R}$ is the Darcy's resistance, p is the pressure, μ is the constant viscosity, Λ_1 is the relaxation time, ρ is the fluid density, $\mathbf{S}$ is the extra stress tensor, T' is the Cauchy stress tensor, $\mathbf{A}_1$ is the first Rivlin–Ericksen tensor, $\mathbf{r}$ is the radiant heating (assumed to be zero), $e = C_pT$ is the specific internal energy, where C_p is the specific heat and T is the temperature, $\mathbf{q} = -k\nabla T$ is the heat flux vector, where k is the constant thermal conductivity, C is the mass concentration, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio and k_1 is the chemical reaction parameter.

3. Formulation of the problem

Let us consider the steady and incompressible flow of Maxwell fluid in an asymmetric channel. The surface is maintained at uniform constant temperature and concentration, see Fig. 1. The flow has significant Soret effect while satisfying creeping flow assumption.

The motion of an incompressible fluid is caused by sinusoidal wave trains propagating with constant speed c along the channel walls as defined by the following pair of equations:

$$\begin{aligned} h_1'(X',t') &= d_1 + a_1\sin\left(\frac{2\pi}{\lambda_1}(X'-ct')\right), \\ h_2'(X',t') &= -d_2 - b_1\sin\left(\frac{2\pi}{\lambda_1}(X'-ct') + \varphi\right). \end{aligned} \tag{9}$$

In Eq. (9), a_1 and b_1 are the waves amplitude, $d_1 + d_2$ is the channel width, λ_1 is the wave length, c is the wave speed, $\varphi(0 \leqslant \varphi \leqslant \pi)$ is the phase difference, X' and Y' are the rectangular coordinates. Moreover, $\varphi = 0$ corresponds to symmetric channel with waves out of phase and for $\varphi = \pi$ the waves are in phase. Further, a_1, b_1, d_1, d_2 and φ satisfy the condition: $a_1^2 + b_1^2 + 2a_1b_1\cos\varphi \leqslant (d_1+d_2)^2$. The wall $Y = h_1'$ is kept at a temperature T_0 and concentration C_0 and the wall $Y = h_2'$ is kept at a temperature T_1 and concentration C_1.

Introducing a wave frame (x', y') moving with velocity c away from the fixed frame (X', Y') by the transformation:

$$\begin{aligned} x' &= X' - ct, \quad y' = Y', \quad u'(x', y') \\ &= U'(X', Y', t') - c, \quad v'(x', y') = V'(X', Y', t'), \end{aligned} \tag{10}$$

where (u', v') are the velocity components in wave frame. The governing equations in the wave frame are given as below:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0, \tag{11}$$

$$\rho\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]u' = -\frac{\partial p'}{\partial x'} + \frac{\partial S'_{xx'}}{\partial x'} + \frac{\partial S'_{xy'}}{\partial y'}, \tag{12}$$

$$\rho\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]v' = -\frac{\partial p'}{\partial y'} + \frac{\partial S'_{xy'}}{\partial x'} + \frac{\partial S'_{yy'}}{\partial y'}, \tag{13}$$

$$\begin{aligned} \rho C_p\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]T &= k\left[\frac{\partial^2 T}{\partial x'^2} + \frac{\partial^2 T}{\partial y'^2}\right] + S'_{xx'}\frac{\partial u'}{\partial x'} + S'_{yy'}\frac{\partial v'}{\partial y'} \\ &\quad + S'_{xy'}\left[\frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'}\right] - \frac{1}{\rho}\frac{\partial q_r}{\partial y}, \end{aligned} \tag{14}$$

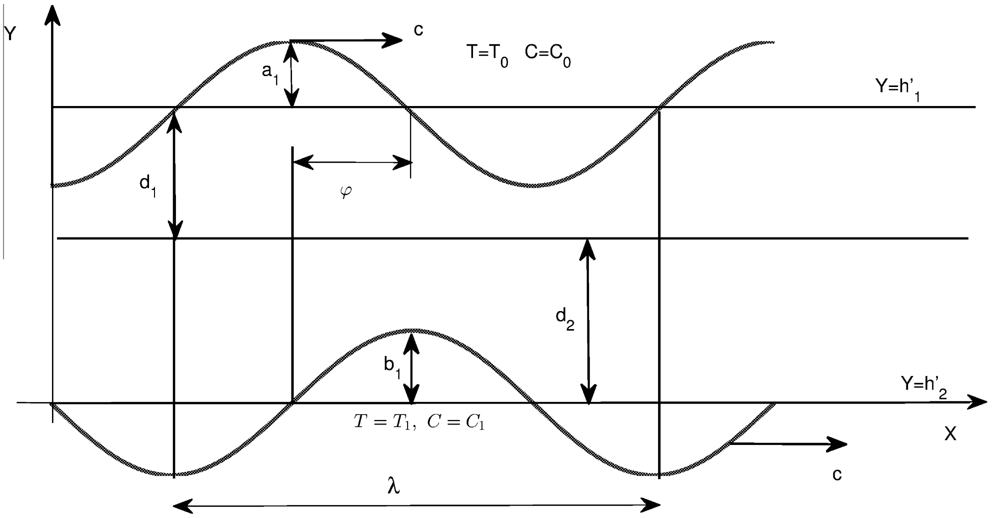

Fig. 1. Physical model of the problem.

$$\left[u'\frac{\partial}{\partial x'}+v'\frac{\partial}{\partial y'}\right]C=D\left[\frac{\partial^2 C}{\partial x'^2}+\frac{\partial^2 C}{\partial y'^2}\right]+\frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial x'^2}+\frac{\partial^2 T}{\partial y'^2}\right]-k_1(C-C_0). \tag{15}$$

$$S'_{xx'}+\Lambda_1\left[\left(u'\frac{\partial}{\partial x'}+v'\frac{\partial}{\partial y'}\right)S'_{xx'}-2\left(\frac{\partial u'}{\partial x'}S'_{xx'}+\frac{\partial u'}{\partial y'}S'_{xy'}\right)\right]=2\mu\frac{\partial u'}{\partial x'}, \tag{16}$$

$$S'_{yy'}+\Lambda_1\left[\left(u'\frac{\partial}{\partial x'}+v'\frac{\partial}{\partial y'}\right)S'_{yy'}-2\left(\frac{\partial v'}{\partial x'}S'_{xy'}+\frac{\partial v'}{\partial y'}S'_{yy'}\right)\right]=2\mu\frac{\partial v'}{\partial y'}, \tag{17}$$

$$S'_{xy'}+\Lambda_1\left[\left(u'\frac{\partial}{\partial x'}+v'\frac{\partial}{\partial y'}\right)S'_{xy'}-\left(\frac{\partial u'}{\partial y'}S'_{xy'}+\frac{\partial v'}{\partial x'}S'_{xx'}\right)\right]=\mu\left[\frac{\partial u'}{\partial y'}+\frac{\partial v'}{\partial x'}\right], \tag{18}$$

We have introduced the following dimensionless quantities:

$$\left.\begin{array}{l}
x=\frac{2\pi x'}{\lambda_1},\quad y=\frac{Y'}{d_1},\quad u=\frac{u'}{c},\quad v=\frac{v'}{c},\quad a=\frac{a_1}{d1},\quad b=\frac{b_1}{d_1},\\
d=\frac{d_2}{d_1},\quad S=\frac{d_1 S'}{\mu c},\quad Re=\frac{\rho c d_1}{\mu},\quad \lambda=\frac{\Lambda_1 c}{d_1},\quad \delta=\frac{2\pi d_1}{\lambda_1},\\
p=\frac{2\pi d_1^2 p'}{\lambda\mu c},\quad h_1=\frac{h'_1}{d_1},\quad h_2=\frac{h'_2}{d_1},\quad u=\frac{\partial\psi}{\partial y},\quad v=-\delta\frac{\partial\psi}{\partial x},\\
\gamma=\frac{T-T_0}{T_1-T_0},\quad \phi=\frac{C-C_0}{C_1-C_0},\quad R=\frac{16\sigma^* T_0^3 d_1^2}{3kK^*},\quad \Pr=\frac{\mu C_p}{k},\\
E=\frac{c^2}{C_p(T_1-T_0)},\quad Br=E\Pr,\quad Sc=\frac{\mu}{\rho D},\quad \chi=\frac{\rho k_1 d_1^2}{\mu}\\
Sr=\frac{\rho(T_1-T_0)DK_T}{\mu T_m(C_1-C_0)}.
\end{array}\right\} \tag{19}$$

Substituting Eq. (19) into Eqs. (11)–(18), we obtain:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{20}$$

$$Re\delta\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right]=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}, \tag{21}$$

$$-\delta^3 Re\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial x}\right]=-\frac{\partial p}{\partial y}+\delta\frac{\partial S_{yy}}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}, \tag{22}$$

$$\delta\Pr Re\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\gamma\right]=\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\gamma+Br\begin{bmatrix}\delta(S_{xx}-S_{yy})\frac{\partial^2\psi}{\partial x\partial y}+\\ \left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)S_{xy}\end{bmatrix}+R\frac{\partial^2\gamma}{\partial y^2}, \tag{23}$$

$$\delta Re\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\phi\right]=\frac{1}{Sc}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\phi+Sr\left[\delta^2\frac{\partial^2\gamma}{\partial x^2}+\frac{\partial^2\gamma}{\partial y^2}\right]-\chi\phi, \tag{24}$$

$$S_{xx}+\lambda\left[\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)S_{xx}-2\left(\delta S_{xx}\frac{\partial^2\psi}{\partial x\partial y}+S_{xy}\frac{\partial^2\psi}{\partial y^2}\right)\right]=2\delta\frac{\partial^2\psi}{\partial x\partial y}, \tag{25}$$

$$S_{yy}+\lambda\left[\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)S_{yy}+2\left(\delta^2 S_{xy}\frac{\partial^2\psi}{\partial x^2}+\delta S_{yy}\frac{\partial^2\psi}{\partial x\partial y}\right)\right]=-2\delta\frac{\partial^2\psi}{\partial x\partial y}, \tag{26}$$

$$S_{xy}+\lambda\left[\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)S_{xy}+\left(\delta^2 S_{xx}\frac{\partial^2\psi}{\partial x^2}-S_{yy}\frac{\partial^2\psi}{\partial y^2}\right)\right]=\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}. \tag{27}$$

By eliminating p from the Eqs. (21) and (22), we get the following equation:

$$\delta Re\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\left(\delta^2\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)\right]$$
$$=\delta^2\frac{\partial^2}{\partial x\partial y}(S_{xx}-S_{yy})+\left(\frac{\partial^2}{\partial y^2}-\delta^2\frac{\partial^2}{\partial x^2}\right)S_{xy}, \tag{28}$$

Thus, the continuity equation is satisfied; whereas ψ is the stream function, f is the flux in the wave frame, $Re=\frac{\rho c d_1}{\mu}$ is the Reynolds number, $\lambda=\frac{\Lambda_1 c}{d_1}$ is the Weissenberg number, $\delta=\frac{2\pi d_1}{\lambda_1}$ is the wave number, $Br = E\Pr$ is the Brinkman number, $\Pr=\frac{\mu C_p}{k}$ is the Prandtl number, $E=\frac{c^2}{C_p(T_1-T_0)}$ is the Eckert number, $Sc=\frac{\mu}{\rho D}$ is the Schmidt number and $Sr=\frac{\rho(T_1-T_0)DK_T}{\mu T_m(C_1-C_0)}$ is the Soret number. The boundary conditions conferred are as follows:

$$\left.\begin{array}{l}
\psi=\frac{f}{2},\quad y=h_1(x)=1+a\sin x,\\
\psi=-\frac{f}{2},\quad y=h_2(x)=-d-b\sin(x+\varphi),\\
\frac{\partial\psi}{\partial y}=-1\quad \text{at } y=h_1\quad \text{and } y=h_2,\\
\gamma=0,\quad \phi=0,\quad \text{at } y=h_1,\\
\gamma=1,\quad \phi=1,\quad \text{at } y=h_2.
\end{array}\right\} \tag{29}$$

The dimensionless forms of $h'_1(X',t')$ and $h'_2(X',t')$ are given by:

$$h_1(x)=1+a\sin x\quad h_2(x)=-d-b\sin(x+\varphi),$$

which satisfy the following condition:

$$a^2+b^2+2ab\cos\varphi\leqslant(1+d)^2,$$

where the respective dimensionless quantities mean flow rate θ and f in the fixed and wave frame are related through the following equation:

$$f=\theta-1-d,$$

while γ and ϕ represent the temperature and mass profiles at the walls.

For creeping flow ($Re\ll 1$), the inertia terms are neglected from the following set of Eqs. (21)–(24) and (28). The Darcy's resistance **R** and chemical reaction parameter k_1 are assumed to be zero which leads to $R\frac{\partial^2\gamma}{\partial y^2}\to 0$ and $\chi\phi\to 0$. Thus, we obtain the following:

$$-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}=0, \tag{30}$$

$$-\frac{\partial p}{\partial y}+\delta\frac{\partial S_{yy}}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}=0, \tag{31}$$

$$\delta^2\frac{\partial^2}{\partial x\partial y}(S_{xx}-S_{yy})+\left(\frac{\partial^2}{\partial y^2}-\delta^2\frac{\partial^2}{\partial x^2}\right)S_{xy}=0, \tag{32}$$

$$\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\gamma+Br\left[\delta(S_{xx}-S_{yy})\frac{\partial^2\psi}{\partial x\partial y}+\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)S_{xy}\right]=0, \tag{33}$$

$$\frac{1}{Sc}\left[\delta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right]+Sr\left[\delta^2\frac{\partial^2\gamma}{\partial x^2}+\frac{\partial^2\gamma}{\partial y^2}\right]=0. \tag{34}$$

4. Perturbation solution

In order to obtain the closed form solution of above mentioned problem, an effort has been made by employing power series expansion in the small parameter δ. This solution should be valid for any arbitrary set of values of all parameters used in mathematical model. Major equations governing the perturbation solution method are as follows:

$$\left.\begin{array}{l}
\psi=\psi_0+\delta\psi_1+\delta^2\psi_2+O(\delta^3),\\
\frac{dp}{dx}=\frac{dp_0}{dx}+\delta\frac{dp_1}{dx}+\delta^2\frac{dp_2}{dx}+O(\delta^3),\\
S_{xx}=S_{0xx}+\delta S_{1xx}+\delta^3 S_{2xx}+O(\delta^3),\\
S_{yy}=S_{0yy}+\delta S_{1yy}+\delta^3 S_{2yy}+O(\delta^3),\\
S_{xy}=S_{0xy}+\delta S_{1xy}+\delta^3 S_{2xy}+O(\delta^3),\\
f=f_0+\delta f_1+\delta^2 f_2+O(\delta^3),\\
p=p_0+\delta p_1+\delta^2 p_2+O(\delta^3),\\
\gamma=\gamma_0+\delta\gamma_1+\delta^2\gamma_2+O(\delta^3),\\
\phi=\phi_0+\delta\phi_1+\delta^2\phi_2+O(\delta^3).
\end{array}\right\} \tag{35}$$

Inserting these equations into Eqs. (25)–(27) and (30)–(34) we can get the following systems.

4.1. Zeroth order system

$$\frac{\partial p_0}{\partial x}=\frac{\partial^3\psi_0}{\partial y^3}, \tag{36}$$

$$\frac{\partial p_0}{\partial y}=0, \tag{37}$$

$$\frac{\partial^4\psi_0}{\partial y^4}=0, \tag{38}$$

$$S_{0yy}=0, \tag{39}$$

$$S_{0xy}=\frac{\partial^2\psi_0}{\partial y^2}, \tag{40}$$

$$S_{0xx}=2\lambda\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2, \tag{41}$$

$$\frac{\partial^2\gamma_0}{\partial y^2}+Br\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2=0, \tag{42}$$

$$\frac{1}{Sc}\frac{\partial^2\phi_0}{\partial y^2}+Sr\left(\frac{\partial^2\gamma_0}{\partial y^2}\right)=0, \tag{43}$$

$$\psi_0=\frac{f_0}{2},\quad \frac{\partial\psi_0}{\partial y}=-1,\quad \gamma_0=0,\quad \phi_0=0,\quad \text{at } y=h_1(x), \tag{44}$$

$$\psi_0=-\frac{f_0}{2},\quad \frac{\partial\psi_0}{\partial y}=-1,\quad \gamma_0=1,\quad \phi_0=1,\quad \text{at } y=h_2(x). \tag{45}$$

4.2. First order system

$$S_{1yy}=-2\frac{\partial^2\psi_0}{\partial x\partial y}, \tag{46}$$

$$S_{1xy}=\frac{\partial^2\psi_1}{\partial y^2}-\lambda\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial^2\psi_0}{\partial y^2}-2\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_0}{\partial x\partial y}\right], \tag{47}$$

$$\begin{aligned}S_{1xx}={}&2\frac{\partial^2\psi_0}{\partial x\partial y}+4\lambda\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}\\&-2\lambda^2\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2\right]\\&-2\lambda^2\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right],\end{aligned} \tag{48}$$

$$\begin{aligned}\frac{\partial p_1}{\partial x}={}&2\lambda\frac{\partial}{\partial x}\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2\right]+\frac{\partial^3\psi_1}{\partial y^3}\\&-\lambda\frac{\partial}{\partial y}\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial^2\psi_0}{\partial y^2}\right]\\&-2\lambda\frac{\partial}{\partial y}\left[\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_0}{\partial x\partial y}\right],\end{aligned} \tag{49}$$

$$\frac{\partial p_1}{\partial y}=0, \tag{50}$$

$$\begin{aligned}-2\lambda\frac{\partial^2}{\partial x\partial y}\left[\left(\frac{\partial\psi_0}{\partial y}\right)^2\right]={}&\frac{\partial^4\psi_1}{\partial y^4}-\lambda\frac{\partial^2}{\partial y^2}\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial^2\psi_0}{\partial y^2}\right]\\&-2\lambda\frac{\partial^2}{\partial y^2}\left[\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_0}{\partial x\partial y}\right],\end{aligned} \tag{51}$$

$$\frac{\partial^2\gamma_1}{\partial y^2}+Br\left[2\frac{\partial^2\psi_1}{\partial y^2}\frac{\partial^2\psi_0}{\partial y^2}-\lambda\left\{\frac{\partial^2\psi_0}{\partial y^2}\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial^2\psi_0}{\partial y^2}\right\}\right]=0, \tag{52}$$

$$\frac{1}{Sc}\frac{\partial^2\phi_1}{\partial y2}+Sr\frac{\partial^2\gamma_1}{\partial y^2}=0, \tag{53}$$

$$\psi_1=\frac{f_1}{2},\quad \frac{\partial\psi_1}{\partial y}=0,\quad \gamma_1=0,\quad \phi_1=0,\quad \text{at } y=h_1(x), \tag{54}$$

$$\psi_1=-\frac{f_1}{2},\quad \frac{\partial\psi_1}{\partial y}=0,\quad \gamma_1=0,\quad \phi_1=0,\quad \text{at } y=h_2(x). \tag{55}$$

4.3. Second order system

$$S_{2xy}=\frac{\partial^2\psi_2}{\partial y^2}-\frac{\partial^2\psi_0}{\partial x^2}-\lambda\left[\left(\frac{\partial\psi_1}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_1}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right]$$

$$
\begin{aligned}
&-\lambda\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_1}{\partial y^2}\right)\right]\\
&+\lambda^2\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)^2\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right]\\
&+4\lambda^2\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\left(\frac{\partial^2\psi_0}{\partial x\partial y}\right)\right]\\
&-4\lambda\left[\left(\frac{\partial^2\psi_0}{\partial x\partial y}\right)\left(\frac{\partial^2\psi_1}{\partial y^2}\right)\right]+4\lambda^2\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\left(\frac{\partial^2\psi_0}{\partial x\partial y}\right)^2\right]\\
&-2\lambda^2\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2\left(\frac{\partial^2\psi_0}{\partial x^2}\right)\right],
\end{aligned}
\tag{56}
$$

$$
\begin{aligned}
\frac{\partial p_2}{\partial x}={}&-2\lambda^2\frac{\partial}{\partial x}\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2\right]+2\frac{\partial}{\partial x}\left[\frac{\partial^2\psi_0}{\partial x\partial y}\right]\\
&-2\lambda^2\frac{\partial}{\partial x}\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right]\\
&+4\lambda\frac{\partial}{\partial x}\left[\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}\right]+\frac{\partial S_{2xy}}{\partial y},
\end{aligned}
\tag{57}
$$

$$
\frac{\partial p_2}{\partial y}=-\frac{\partial}{\partial y}\left[\frac{\partial^2\psi_0}{\partial x\partial y}\right],
\tag{58}
$$

$$
\begin{aligned}
&2\lambda^2\frac{\partial^2}{\partial x\partial y}\left[\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2\right]-3\frac{\partial^2}{\partial x\partial y}\left[\frac{\partial^2\psi_0}{\partial x\partial y}\right]\\
&+2\lambda^2\frac{\partial^2}{\partial x\partial y}\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\left(\frac{\partial\psi_0}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right]\\
&-4\lambda\frac{\partial^2}{\partial x\partial y}\left[\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}\right]=\frac{\partial^2 S_{2xy}}{\partial y^2},
\end{aligned}
\tag{59}
$$

$$
\left[\frac{\partial^2\gamma_0}{\partial x^2}+\frac{\partial^2\gamma_2}{\partial y^2}\right]+Br\begin{pmatrix} S_{1xx}\frac{\partial^2\psi_0}{\partial x\partial y}+S_{0xx}\frac{\partial^2\psi_1}{\partial x\partial y}-S_{0yy}\frac{\partial^2\psi_1}{\partial x\partial y}-S_{1yy}\frac{\partial^2\psi_0}{\partial x\partial y}\\ +\frac{\partial^2\psi_0}{\partial y^2}S_{2xy}+\frac{\partial^2\psi_1}{\partial y^2}S_{1xy}+\frac{\partial^2\psi_2}{\partial y^2}S_{0xy}-\frac{\partial^2\psi_0}{\partial x^2}S_{0xy}\end{pmatrix}=0,
\tag{60}
$$

$$
\frac{1}{Sc}\left[\frac{\partial^2\gamma_0}{\partial x^2}+\frac{\partial^2\phi_2}{\partial y2}\right]+Sr\left[\frac{\partial^2\gamma_0}{\partial x^2}+\frac{\partial^2\gamma_2}{\partial y2}\right]=0,
\tag{61}
$$

$$
\psi_2=\frac{f_2}{2},\quad \frac{\partial\psi_2}{\partial y}=0,\quad \gamma_2=0,\quad \phi_2=0,\quad \text{at } y=h_1(x),
\tag{62}
$$

$$
\psi_2=-\frac{f_2}{2},\quad \frac{\partial\psi_2}{\partial y}=0,\quad \gamma_2=0,\quad \phi_2=0,\quad \text{at } y=h_2(x).
\tag{63}
$$

After solving the above three systems (Zero, First and Second order), we get the following solutions:

4.4. Zeroth order solution

$$
\begin{aligned}
&\psi_0=L_1y^3+L_2y^2+L_3y+L_4,\\
&u_0=3L_1y^2+2L_2y+L_3,\\
&\gamma_0=P_1y^4+P_2y^3+P_3y^2+P_4y+P_5,\\
&\phi_0=B_1y^4+B_2y^3+B_3y^2+B_4y+B_5,\\
&\frac{dp_0}{dx}=\frac{-12(F_0+h_1-h_2)}{(h_1-h_2)^3},\\
&\Delta P_{\lambda_0}=\int_0^{2\pi}\frac{dp_0}{dx}dx,\\
&F_{\lambda_{01}}=\int_0^{2\pi}-h_1^2(\frac{dp_0}{dx})dx,\quad F_{\lambda_{02}}=\int_0^{2\pi}-h_2^2\left(\frac{dp_0}{dx}\right)dx.
\end{aligned}
$$

4.5. First order solution

$$
\begin{aligned}
&\psi_1=G_1y^3+G_2y^2+G_3y+G_4,\\
&u_1=3G_1y^2+2G_2y+G_3,\\
&\frac{dp_1}{dx}=6G_1+4L_2L_{2x}\lambda-6L_{1x}L_3\lambda-6L_1L_{3x}\lambda,\\
&\gamma_1=R_1y^6+R_2y^5+R_3y^4+R_4y^3+R_5y^2+R_6y+R_7,\\
&\phi_1=S_1y^6+S_2y^5+S_3y^4+S_4y^3+S_5y^2+S_6y+S_7,\\
&\Delta P_{\lambda_1}=\int_0^{2\pi}\frac{dp_1}{dx}dx,\\
&F_{\lambda_{11}}=\int_0^{2\pi}-h_1^2(\frac{dp_1}{dx})dx,\quad F_{\lambda_{12}}=\int_0^{2\pi}-h_2^2\left(\frac{dp_1}{dx}\right)dx.
\end{aligned}
$$

4.6. Second order solution

$$
\begin{aligned}
&\psi_2=K_1y^8+K_2y^7+K_3y^6+K_4y^5+K_5y^4+K_6y^3+K_7y^2+K_8y+K_9,\\
&u_2=8K_1y^7+7K_2y^6+6K_3y^5+5K_4y^4+4K_5y^3+3K_6y^2+2K_7y+K_8,\\
&\frac{dp_2}{dx}=K_{10}y^5+K_{11}y^4+K_{12}y^3+K_{13}y^2+K_{14}y+K_{15},\\
&\gamma_2=C_1y^8+C_2y^7+C_3y^6+C_4y^5+C_5y^4+C_6y^3+C_7y^2+C_8y+C_9,\\
&\phi_2=C_{10}y^8+C_{11}y^7+C_{12}y^6+C_{13}y^5+C_{14}y^4+C_{15}y^3+C_{16}y^2+C_{17}y+C_{18},\\
&\Delta P_{\lambda_2}=\int_0^{2\pi}\frac{dp_2}{dx}dx,\\
&F_{\lambda_{21}}=\int_0^{2\pi}-h_1^2\left(\frac{dp_2}{dx}\right)dx,\quad F_{\lambda_{22}}=\int_0^{2\pi}-h_2^2\left(\frac{dp_2}{dx}\right)dx,
\end{aligned}
$$

All of the constants involved in Zero, First and Second order solutions, as given above, are provided in Appendix.

The expressions of ΔP_λ,dp/dx,γ,ϕ and F_λ up to $O(\delta^2)$ are denoted by $\Delta P_\lambda^{(2)}, dp^{(2)}/dx, \gamma^{(2)}, \phi^{(2)}$ and $F_\lambda^{(2)}$. Mathematically we can write:

$$
\begin{aligned}
&\Delta P_\lambda^{(2)}=\Delta P_{\lambda_0}+\delta\Delta P_{\lambda_1}+\delta\Delta P_{\lambda_2},\\
&dp^{(2)}/dx=\frac{dp_0}{dx}+\delta\frac{dp_1}{dx}+\delta^2\frac{dp_2}{dx},\\
&\gamma^{(2)}=\gamma_0+\delta\gamma_1+\delta^2\gamma_2,\\
&\phi^{(2)}=\phi_0+\delta\phi_1+\delta^2\phi_2,\\
&F_\lambda^{(2)}=F_{\lambda_0}+\delta F_{\lambda_1}+\delta^2F_{\lambda_2}
\end{aligned}
$$

and the final expressions for the pressure rise per wavelength, pressure gradient, temperature, mass and frictional forces can be obtained from Eqs. of zeroth to second order solutions.

5. Discussion

In this section we have discussed our results, namely: pumping characteristics, heat characteristics, mass characteristics, behavior of velocity and trapping. Further, in the next five subsections, we have presented and analyzed the behavior of the solutions for stream function (ψ), velocity (u), temperature ($\gamma^{(2)}$), mass ($\phi^{(2)}$), pressure gradient ($dp^{(2)}/dx$), pressure rise per wave length ($\Delta P_\lambda^{(2)}$) and frictional force ($F_\lambda^{(2)}$) for the several values of wave number (δ), Weissenberg number (λ), phase (φ), Brinkman number (Br), Schmidt number (Sc) and Soret number (Sr).

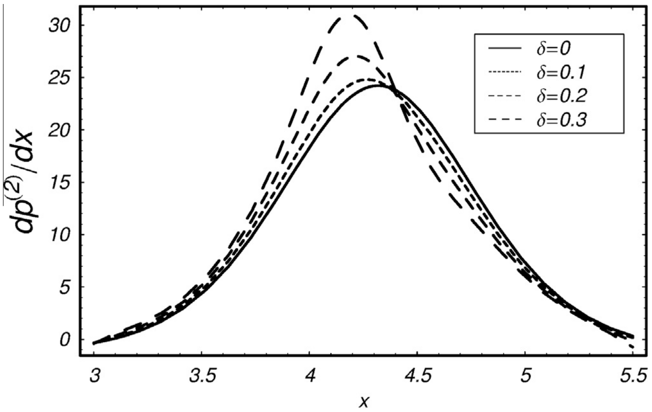

Fig. 2. Influence of δ on $dp^{(2)}/dx$ when $a = 0.5$, $b = 0.5$, $d = 0.6$, $\theta = 0.3$, $\varphi = \pi/4$, and $\lambda = 0.1$.

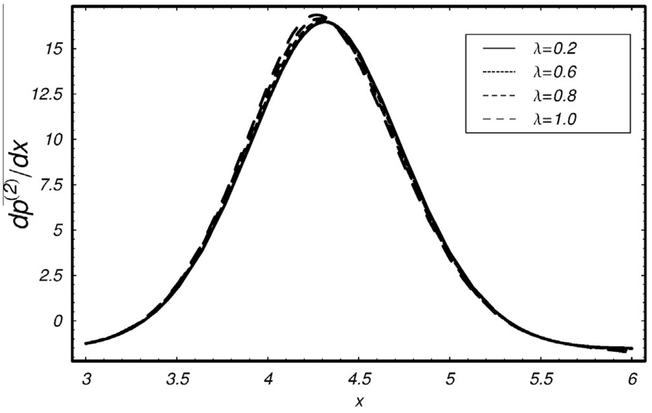

Fig. 3. Influence of λ on $dp^{(2)}/dx$ when $a = 0.5$, $b = 0.5$, $d = 0.6$, $\theta = 0.5$, $\varphi = \pi/4$, and $\delta = 0.01$.

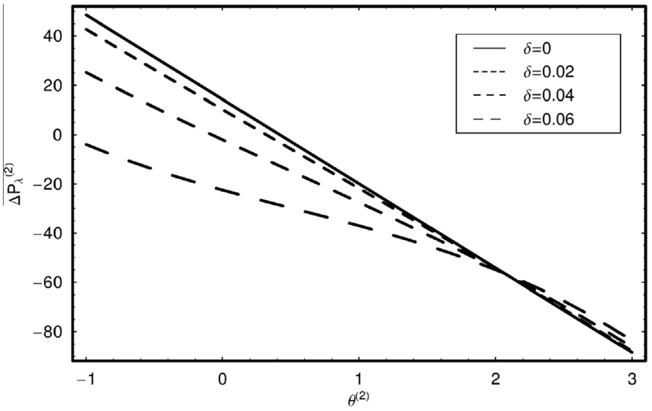

Fig. 4. Influence of δ on $\Delta P_\lambda^{(2)}$ when $a = 0.3$, $b = 0.5$, $d = 0.4$, $\varphi = 0.1$, $\lambda = 0.1$.

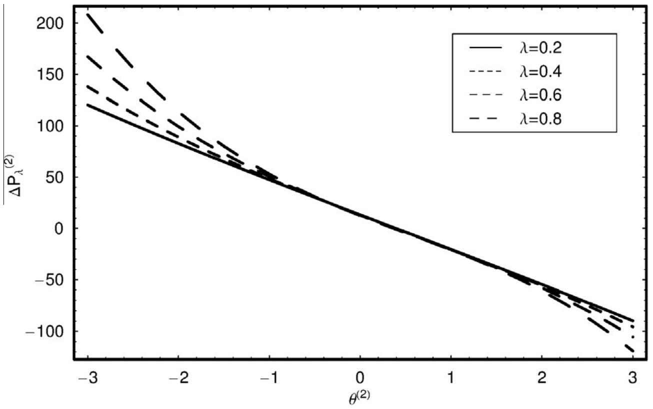

Fig. 5. Influence of λ on $\Delta P_\lambda^{(2)}$ when $a = 0.3$, $b = 0.3$, $d = 0.6$, $\varphi = 0.1$, $\delta = 0.01$.

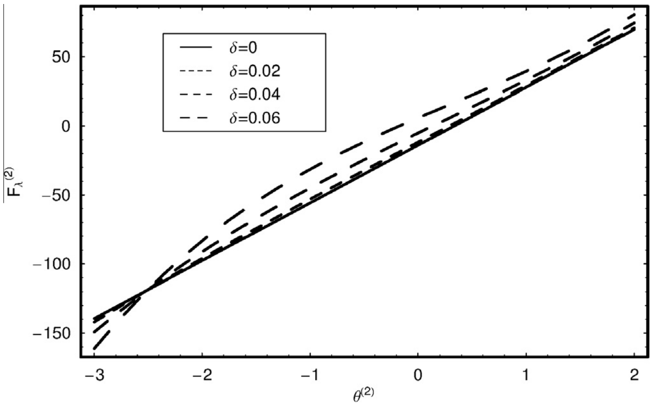

Fig. 6. Influence of δ on $F_\lambda^{(2)}$ at the upper wall $h_1(x)$ when $a = 0.3$, $d = 0.6$, $b = 0.4$, $\varphi = 0.1$, $\lambda = 0.1$.

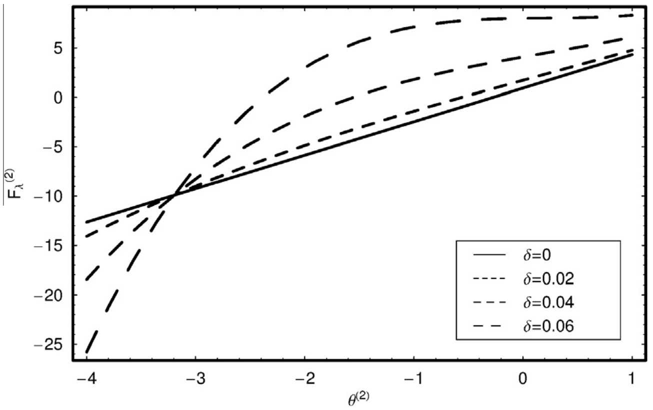

Fig. 7. Influence of δ on $F_\lambda^{(2)}$ at the lower wall $h_2(x)$ when $a = 0.3$, $d = 0.6$, $b = 0.3$, $\varphi = 0.1$, $\lambda = 0.1$.

5.1. Pumping characteristics

This subsection illustrates the behavior of emerging parameters δ and λ on $dp^{(2)}/dx, \Delta P_\lambda^{(2)}$ and $F_\lambda^{(2)}$. In Fig. 2 the variation of $dp^{(2)}/dx$ versus x is shown for different values of δ by keeping the other parameters at fixed values. We can observe that pressure gradient is relatively small in the wider part of the channel. The flow can easily pass without imposition of large pressure gradient. Inspite of being in the narrow part of the channel there is a need of much pressure gradient to maintain the flux to pass through it, especially near $x = 4.4$. In short $dp^{(2)}/dx$ in viscous fluid is less than Maxwell fluid. The behavior of λ in Fig. 3 is similar to the behavior of δ in Fig. 2.

The dimensionless pressure rise per wave length versus variation of time-averaged flux θ has been plotted in Figs. 4 and 5. Here the upper right-hand quadrant (I) denotes the region of peristalsis pumping, where $\theta^{(2)} > 0$ (positive pumping) and $\Delta P_\lambda^{(2)} > 0$ (adverse pressure gradient). Quadrant (II), where $\Delta P_\lambda^{(2)} < 0$ (favorable

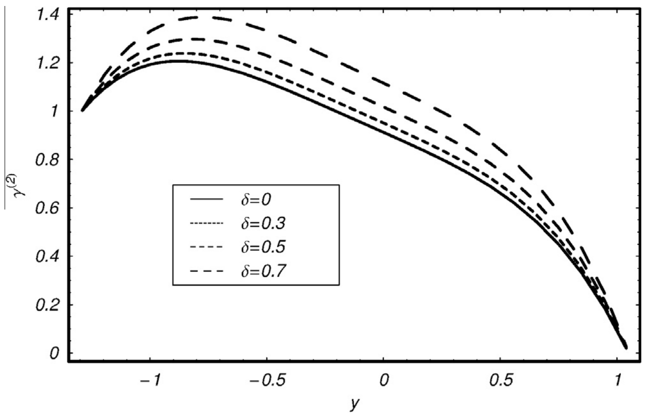

Fig. 8. Influence of δ on $\gamma^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$.

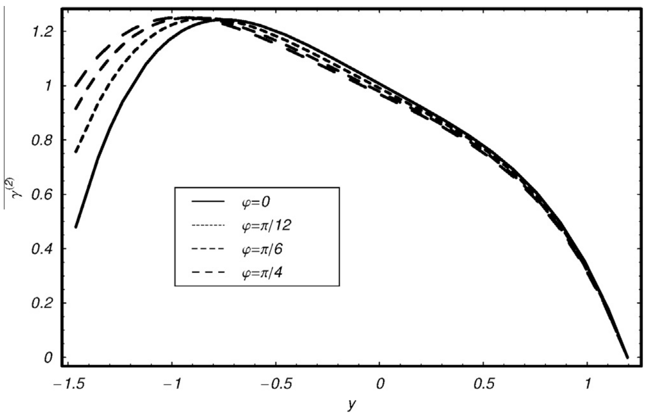

Fig. 11. Influence of φ on $\gamma^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\delta = 0.1$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$.

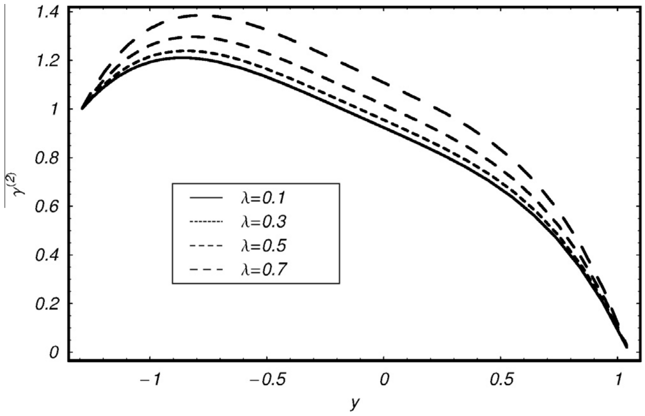

Fig. 9. Influence of λ on $\gamma^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\delta = 0.5$.

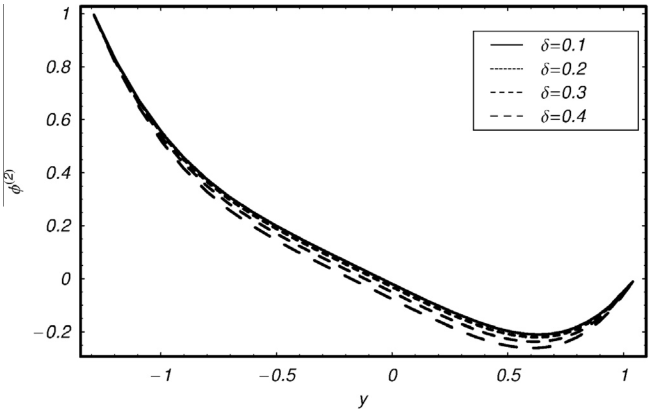

Fig. 12. Influence of δ on $\phi^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$, $Sc = 1$, $Sr = 1$.

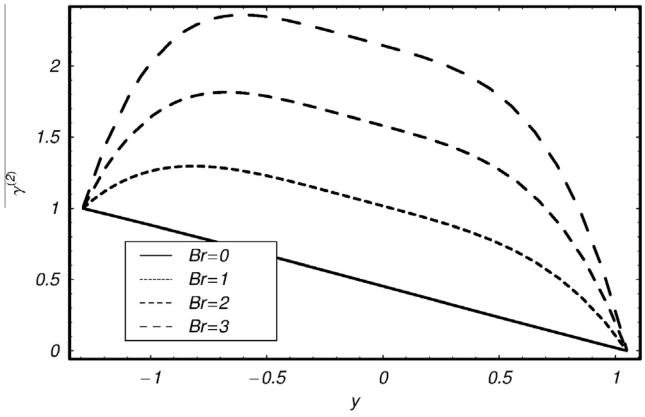

Fig. 10. Influence of Br on $\gamma^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $\delta = 0.5$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$.

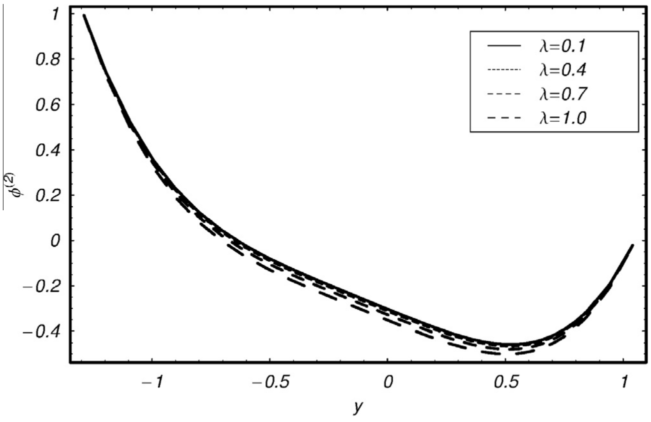

Fig. 13. Influence of λ on $\phi^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\delta = 0.1$, $Sc = 1$, $Sr = 1$.

pressure gradient) and $\theta^{(2)} > 0$ (positive pumping), is designated as augmented flow (copumping region). Quadrant (IV), such that $\Delta P_{\lambda}^{(2)} > 0$ (adverse pressure gradient) and $\theta^{(2)} < 0$, is called retrograde or backward pumping. The flow is opposite to the direction of the peristaltic motion, and there is no flow in the last (Quadrant (III)). Fig. 4 show that, there is an inversely linear relation between $\Delta P_{\lambda}^{(2)}$ and $\theta^{(2)}$, i.e. for the adverse pressure gradient and for the free pumping, the pumping decreases with the increase of δ. On the other hand, in the copumping region the pumping increases with the increase of δ. In Fig. 5, we can say that for the adverse pressure gradient and for the free pumping, the pumping increases with the increase of λ. However, in the copumping region the pumping is decreasing.

The frictional forces $F_{\lambda}^{(2)}$ at the upper wall ($y = h_1(x)$) of the channel with dimensionless time mean flow $\theta^{(2)}$ is plotted in Fig. 6. This shows that there exists a critical value of $\theta^{(2)}$ below which $F_{\lambda}^{(2)}$

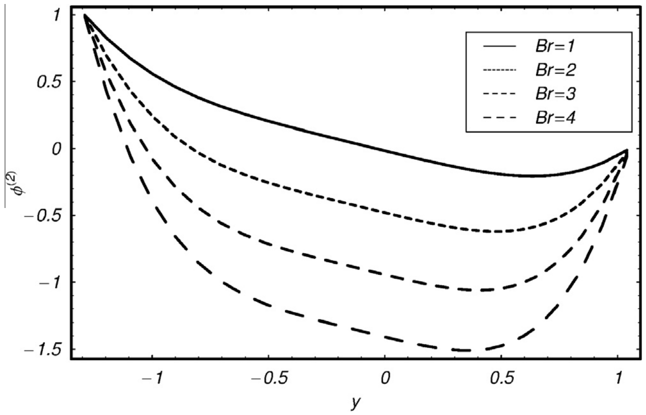

Fig. 14. Influence of Br on $\phi^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $\delta = 0.01$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$, $Sc = 1$, $Sr = 1$.

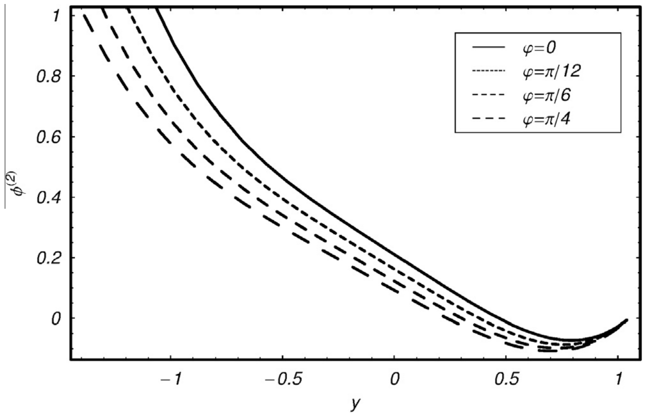

Fig. 15. Influence of φ on $\phi^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\delta = 0.1$, $Br = 1$, $\theta = 1.2$, $x = 0.1$, $\lambda = 0.5$, $Sc = 1$, $Sr = 1$.

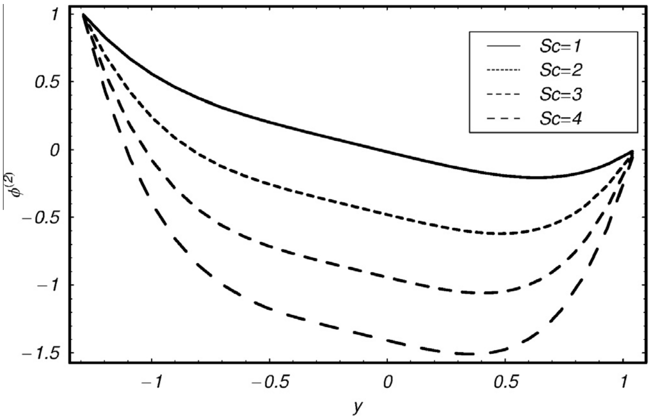

Fig. 16. Influence of Sc on $\phi^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$, $\delta = 0.01$, $Sr = 1$.

resists the flow and an increase in δ causes an increase in $F_\lambda^{(1)}$. Fig. 7 shows the influence of δ on $F_\lambda^{(2)}$ at the lower wall ($y = h_2(x)$) of the channel with $\theta^{(2)}$. We note that $F_\lambda^{(1)}$ assists the flow for all $\theta^{(1)} \geqslant 0$ and δ plays the similar role for the variation of $F_\lambda^{(1)}$ at the lower and upper walls of the channel.

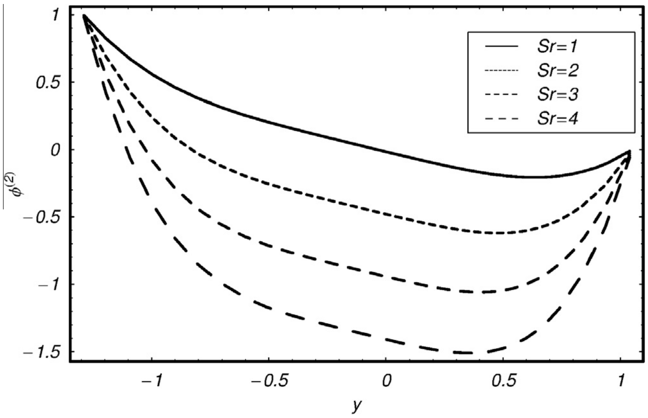

Fig. 17. Influence of Sr on $\phi^{(2)}$ when $a = 0.5$, $b = 0.5$, $d = 1$, $\varphi = \pi/6$, $Br = 1$, $\theta = 1.5$, $x = 0.1$, $\lambda = 0.5$, $Sc = 1$, $\delta = 0.01$.

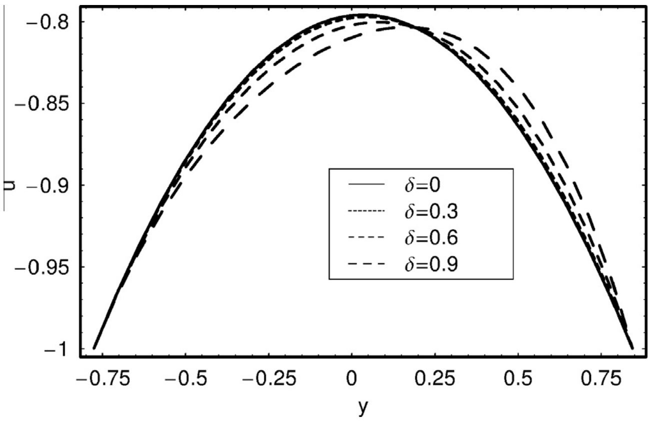

Fig. 18. Influence of δ on u when $a = 0.4$, $b = 0.3$, $d = 0.5$, $\varphi = \pi/2$, $\theta = 0.1$, $x = -0.4$, $\lambda = 0.1$.

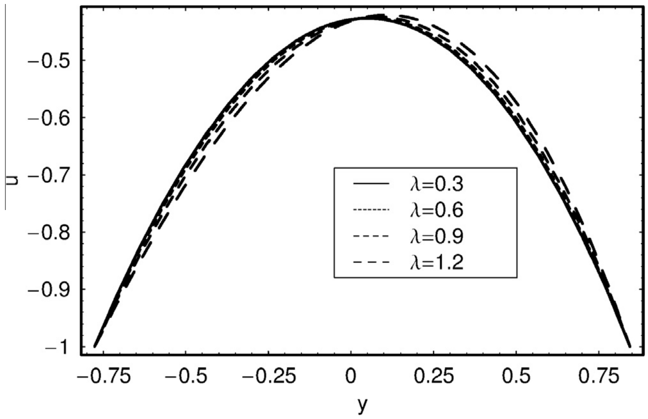

Fig. 19. Influence of λ on u when $a = 0.4$, $b = 0.3$, $d = 0.5$, $\varphi = \pi/2$, $\theta = 0.5$, $x = -0.4$, $\delta = 0.5$.

5.2. Heat characteristics

Effect of heat transfer on peristalsis is shown in Figs. 8–11. In Fig. 8 and 9, we have observed the effects of δ and λ on the temperature profile $\gamma^{(2)}$ while keeping the other parameters at fixed values. These figures indicate that the temperature profiles are

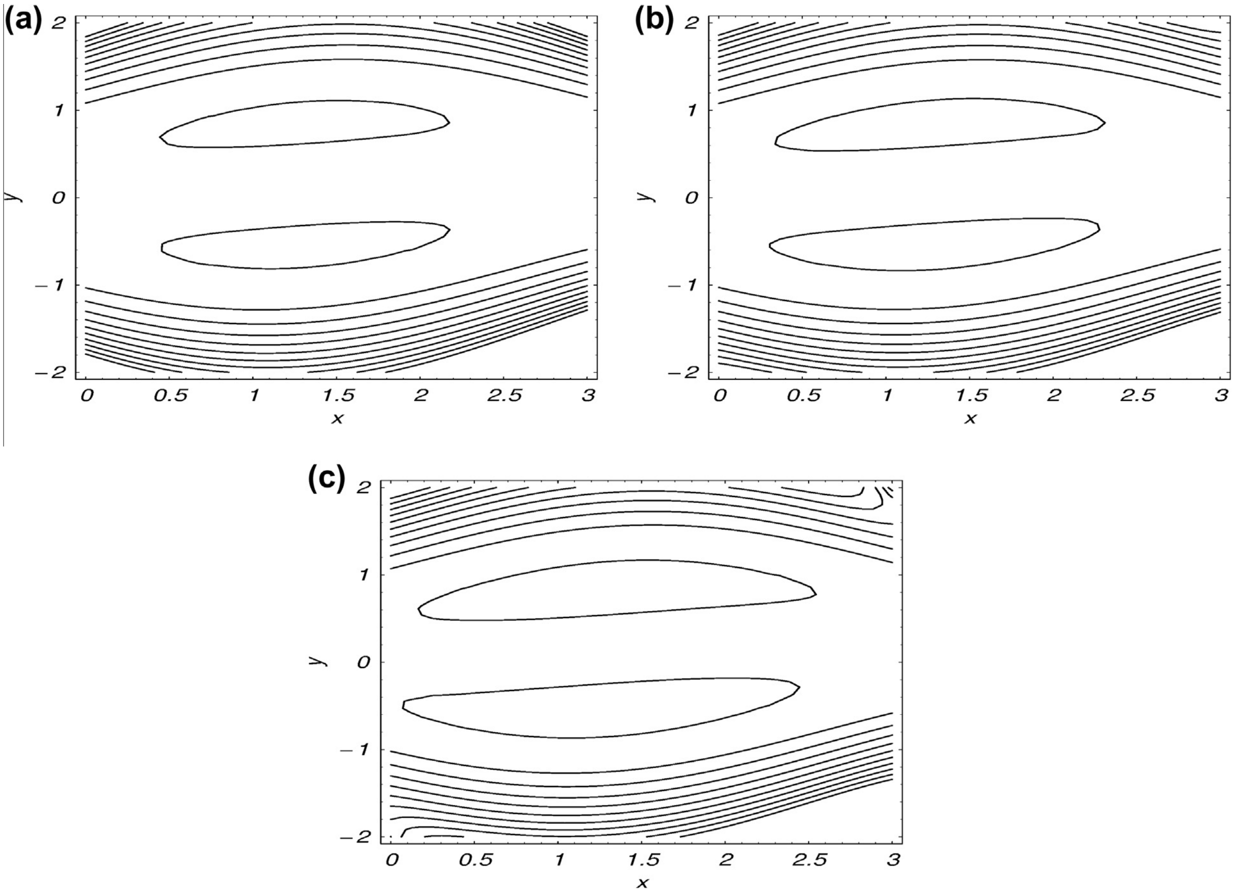

Fig. 20. Influence of δ on ψ with $a = 0.5$, $b = 0.5$, $d = 0.7$, $\theta = 1.55$, $\lambda = 1.5$, $\varphi = \pi/6$. (a) $\delta = 0$ (b) $\delta = 0.06$ (c) $\delta = 0.09$.

almost parabolic and temperature increases with the increase of δ and λ. Fig. 10 depicts the effect of *Br* on the temperature profile. One can observe that the temperature profile is an increasing function of *Br*. Also, the Fig. 10 shows that the temperature profiles are almost parabolic except when $Br = 0$. In Fig. 11, the effect of φ on the temperature profile θ has been observed. No considerable variation occurs near the wall $y = h_1$, but the amplitude of the temperature near the wall $y = h_2$ of the channel has increased.

5.3. Mass characteristics

Influence of mass transfer on peristalsis is shown in Figs. 12–17. The Figs. 12 and 13 depict the similar behavior as with the increase of δ and λ the concentration is decreased. Fig. 14 illustrates that concentration is a decreasing function of the *Br*. The concentration profiles $\phi^{(2)}$ are almost parabolic. Fig. 15 gives the effect of φ on the concentration when the other parameters are fixed. It can be predict that concentration is a decreasing function of φ near the both walls. Figs. 16 and 17 illustrate that concentration is a decreasing function of *Sc* and *Sr*.

5.4. Behavior of velocity

The variations of the small δ and λ on the longitudinal velocity u in the asymmetric channel have been shown in this subsection. Fig. 18 gives the result that an increase in δ decreases the magnitude of u near the boundaries. However, at the center of the channel the role of wave number is quite opposite to that of near the boundaries. Fig. 19 gives the same behavior for the λ as for the δ.

5.5. Trapping

In this subsection, the variation of δ and λ are shown in Fig. 20 and Fig. 21. Fig. 20 is plotted for the δ. Panel (a) is for the viscous fluid and bolus is symmetric about the center line. Panels (b) and (c) show that the bolus increases with an increase of wave number. Fig. 21 shows the trapping for the λ. In panel (a) the bolus is symmetric about the center line and panels (b), (c) and (d) indicate that the bolus increases with an increase in λ.

6. Conclusion

The effects of creeping flow on the peristaltic transport of Maxwell fluid in an asymmetric channel with heat and mass transfer have been analyzed. Numerical and analytical solutions have also been developed for the stream function, velocity, temperature, mass, frictional forces, pressure gradient, pressure rise per wave length and rate of heat and mass transfer. The final results obtained are as follows:

- For the creeping flow, we have result that is similar to that for the viscous case up to first order but afterwards in case of second order we get the results for Maxwell fluid Hayat et al. [26].
- The peristaltic pumping rate decreases by increasing the wave number and in the copumping region the pumping rate increases by increasing the wave number.
- The magnitude of longitudinal velocity at the boundaries is decreasing function of wave number and at the center of the channel is an increasing function of the wave number.

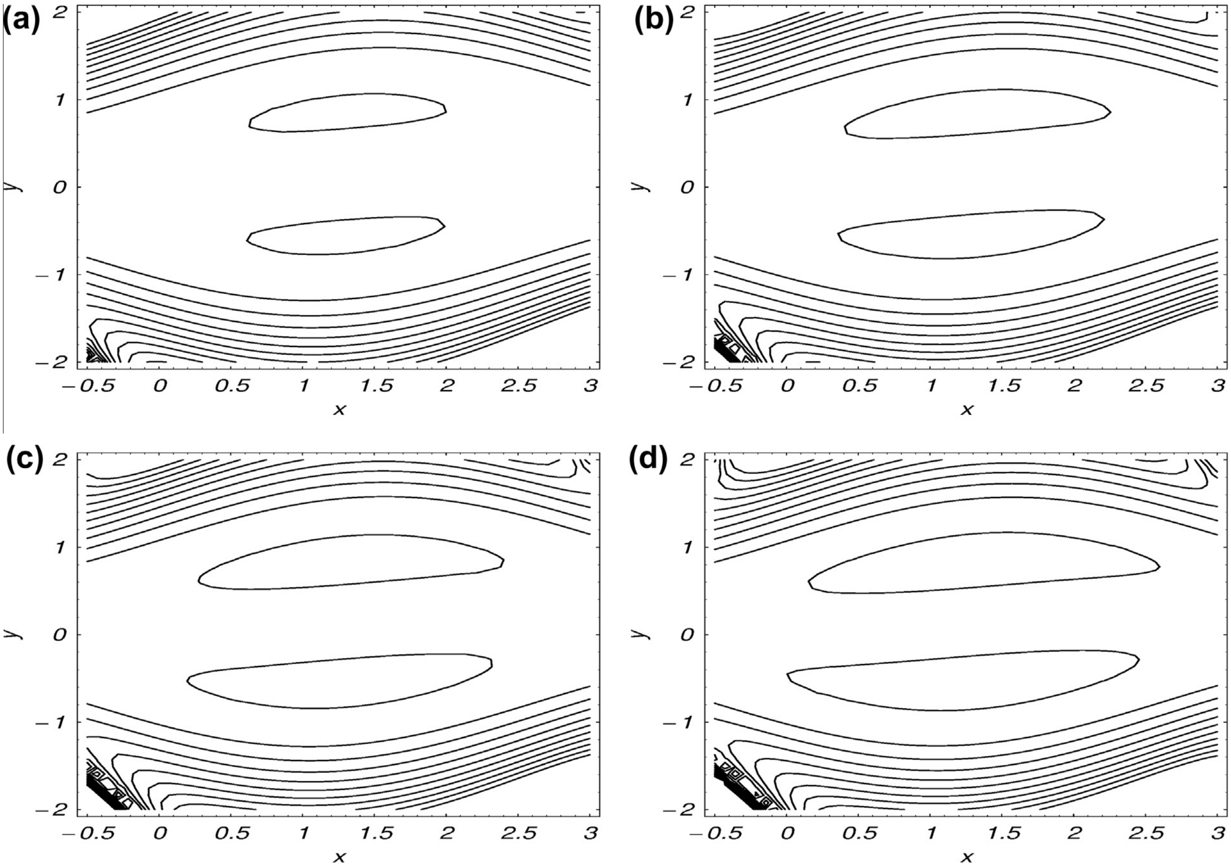

Fig. 21. Influence of λ on ψ with $a = 0.5$, $b = 0.5$, $d = 0.7$, $\theta = 1.55$, $\delta = 0.09$, $\varphi = \pi/6$. (a)$\lambda = 1$, (b)$\lambda = 1.3$, (c)$\lambda = 1.5$, (d)$\lambda = 1.7$.

- The size of the trapped bolus increases by increasing the wave number.
- The temperature profile is increased as the values of the wave number, Weissenberg number, Brinkman number and phase are increased.
- The concentration profile is decreased as the values of wave number, Weissenberg number, Brinkman number, Schmidt number, Soret number and phase are increased.

Appendix A

$$L_1 = \frac{-2(f_0 + h_1 - h_2)}{(h_1 - h_2)^3}, \quad L_2 = \frac{3(f_0 + h_1 - h_2)(h_1 + h_2)}{(h_1 - h_2)^3},$$

$$L_3 = \frac{-(h_1^3 + 3h_1^2 h_2 + 3h_1(2f_0 - h_2)h_2 - h_2^3)}{(h_1 - h_2)^3},$$

$$L_4 = \frac{-(h_1 + h_2)(2h_1 h_2(-h_1 + h_2) + f_0(h_1^2 - 4h_1 h_2 + h_2^2))}{2(h_1 - h_2)^3},$$

$$P_1 = -3BrL_1^2, \quad P_2 = -4BrL_1L_2, \quad P_3 = -2BrL_2^2,$$

$$P_4 = \frac{1}{(h_1 - h_2)}(-1 + Br(3h_1^4L_1^2 + 4h_1^3L_1L_2 + 2h_1^2L_2^2 - h_2^2(3h_2^2L_1^2 + 4h_2L_1L_2 + 2L_2^2))),$$

$$P_5 = \frac{1}{(h_1 - h_2)}(h_1(1 + Brh_2(-3h_1^3L_1^2 - 4h_1^2L_1L_2 - 2h_1L_2^2 + h_2(3h_2^2L_1^2 + 4h_2L_1L_2 + 2L_2^2)))),$$

$$B_1 = 3BrL_1^2ScSr, \quad B_2 = 4BrL_1L_2ScSr, \quad B_3 = 2BrL_2^2ScSr,$$

$$B_4 = -\frac{1}{(h_1 - h_2)}(1 + Br(3h_1^4L_1^2 + 4h_1^3L_1L_2 + 2h_1^2L_2^2 - h_2^2(3h_2^2L_1^2 + 4h_2L_1L_2 + 2L_2^2))ScSr),$$

$$B_5 = \frac{1}{(h_1 - h_2)}(h_1(1 + Br(h_1 - h_2)h_2(3h_1^2L_1^2 + 3h_2^2L_1^2 + 4h_2L_1L_2 + 2L_2^2 + h_1L_1(3h_2L_1 + 4L_2))ScSr)),$$

$$G_1 = \frac{-2f_1}{(h_1 - h_2)^3}, \quad G_2 = \frac{3f_1(h_1 + h_2)}{(h_1 - h_2)^3}, \quad G3 = -\frac{6f_1h_1h_2}{(h_1 - h_2)^3},$$

$$G_4 = \frac{-f_1(h_1^3 - 3h_1^2h_2 - 3h_1h_2^2 + h_2^3)}{2(h_1 - h_2)^3}$$

$$K_1 = -\frac{36}{7}L_1L_{1xx}L_{1x}\lambda^2, \quad K_2 = -\frac{1}{30}(288L_1^2L_{1xx}\lambda^2 - 90L_1L_{1xx}L_{1x}^2\lambda^2 + 72L_1L_{1x}L_{2xx}\lambda^2 + 72L_{1xx}L_{1x}L_2\lambda^2 + 216L_1L_{1xx}L_{2x}\lambda^2),$$

$$\begin{aligned}K_3 = &-\frac{1}{15}(72L_1L_{1xx}L_2\lambda^2 - 36L_{1xx}L_{1x}L_2\lambda^2 - 18L_{1x}^2L_2\lambda^2 + 216L_1^2L_{2xx}\lambda_2\\ &- 54L_1L_{1x}L_{2xx}\lambda^2 + 18L_{1x}L_2L_{2xx}\lambda^2 - 72L_1L_{1x}L_{2x}\lambda_2 + 54L_{1xx}L_2L_{2x}\lambda^2\\ &+ 54L_1L_{2xx}L_{2x}\lambda^2 + 162L_1L_{1xx}L_{3x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_4 = &-\frac{1}{6}(72G_{1x}L_1 + 3L_{1xx} + 72G_1L_{1x} - 18G_{1x}L_1\lambda - 126G_1L_{1x}\lambda\\ &+ 324L_1^2L_{1xx}\lambda^2 - 12L_{1xx}L_2^3\lambda^2 + 72L_1L_2L_{2xx}\lambda^2 - 18L_{1x}L_2L_{2xx}\lambda^2\\ &- 12L_{1x}L_2L_{2x}\lambda_2 + 12L_2L_{2xx}L_{2x}\lambda^2 + 18L_1L_{2x}^2 + 18L_1L_{2x}^2\lambda^2\\ &- 9L_{1xx}L_{1x}L_3\lambda^2 + 108L_1^2L_{3xx}\lambda^2 - 27L_1L_{1x}L_{3xx}\lambda^2 - 108L_1L_{1x}L_{3x}\lambda^2\\ &+ 12L_2L_{2xx}L_{2x}\lambda^2 + 36L_1L_{2xx}L_{3x}\lambda^2 + 36L_1L_{2xx}L_{3x}\lambda^2\\ &+ 108L_1L_{1xx}L_{4x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_5 = &-4(12G_{2x}L_1 + 12G_{1x}L_2 + L_{2xx} + 12G_1L_{2x} - 6G_{1x}L_2\lambda - 24G_1L_{2x}\lambda\\ &+ 72L_1L_{1xx}L_2\lambda^2 + 2L_2L_{2x}^2\lambda_2 + 6L_2L_{2xx}L_{3x}\lambda^2 + 6L_1L_{2x}L_{3x}\lambda^2\\ &+ 18L_1^2L_{4xx}\lambda^2 - 9L_1L_{1x}L_{4xx}\lambda^2 - 90L_1L_{1x}L_{4x}\lambda^2 + 18L_{1xx}L_2L_{4x}\lambda^2\\ &+ 18L_1L_{2xx}L_{4x}\lambda^2 - 6G_2L_{1x}(-2 + 3\lambda) - 6L_{1xx}L_2L_3\lambda^2 + 6L_1L_{2xx}L_3\lambda^2\\ &+ 36L_1^2L_{2xx}\lambda^2 - 3L_{1x}L_{2xx}L_3\lambda^2 + 30L_1L_2L_{3xx}\lambda^2 - 6L_{1x}L_{3xx}\lambda^2\\ &- 12L_{1x}L_2L_{3x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_6 = &-\frac{1}{420(h_1 - h_2)^3}(5040f_2 + (h_1 - h_2)^3(28K_3h_1^3 + 15K_2h_1^4\\ &+ 9K_1h_1^5 + 42K_3h_1^2h_2 + 24K_2h_1^3h_2 + 15K_1h_1^4h_2 + 42K_3h_1h_2^2\\ &+ 27K_2h_1^2h_2^2 + 18K_1h_1^3h_2^2 + 28K_3h_2^3 + 24K_2h_1h_2^3 + 18K_1h_1^2h_2^3\\ &+ 15K_2h_2^4 + 15K_1h_1h_2^4 + 9K_1h_2^5 + 210K_5(h_1 + h_2) + 21K_4(3h_1^2\\ &+ 4h_1h_2 + 3h_2^2))),\end{aligned}$$

$$\begin{aligned}K_7 = &\frac{1}{840(h_1 - h_2)^3}(5040f_2(h_1 + h_2) + (h_1 - h_2)^3(14K_3h_1^4 + 8K_2h_1^5\\ &+ 5K_1h_1^6 + 56K_3h_1^3h_2 + 20K_1h_1^5h_2 + 70K_3h_1^2h_2^2 + 44K_2h_1^3h_2^2\\ &+ 29K_1h_1^4h_2^2 + 56K_3h_1h_2^3 + 44h_1^2h_2^3 + 32K_1h_1^3h_2^3 + 14K_3h_2^4\\ &+ 32K_2h_1h_2^4 + 29K_1h_1^2h_2^4 + 8K_2h_2^5 + 20K_1h_1h_2^5 + 5K_1h_2^6\\ &+ 70K_5(h_1^2 + 4h_1h_2 + h_2^2) + 32K_2h_1^4h_2 + 28K_4(h_1^3 + 4h_1^2h_2\\ &+ 4h_1h_2^2 + h_2^3))),\end{aligned}$$

$$\begin{aligned}K_8 = &-\frac{1}{840(h_1 - h_2)^3}(h_1h_2(5040f_2 + (h_1 - h_2)^3(14K_3h_1^3 + 8K_2h_1^4\\ &+ 5K_1h_1^5 + 28K_3h_1^2h_2 + 17K_2h_1^3h_2 + 11K_1h_1^4h_2 + 28K_3h_1h_2^2\\ &+ 20K_2h_1^2h_2^2 + 14K_1h_1^3h_2^2 + 14K_3h_2^3 + 17K_2h_1h_2^3 + 14K_1h_1^2h_2^3\\ &+ 8K_2h_2^4 + 11K_1h_1h24 + 5K1h25 + 70K5(h1 + h2)\\ &+ 7K4(4h12 + 7h1h2 + 4h22)))),\end{aligned}$$

$$\begin{aligned}K_9 = &\frac{1}{5040(h_1 - h_2)^3}(-2520f_2(h_1^3 - 3h_1^2h_2 - 3h_1h_2^2 + h_2^3)\\ &+ h_1^2(h_1 - h_2)3h_2^2(210K_5 + 42K_3h_1^2 + 24K_2h_1^3 + 15K_1h_1^4\\ &+ 56K_3h_1h_2 + 36K_2h_1^2h_2 + 24K_1h_1^3h_2 + 42K_3h_2^2 + 42K_3h_2^2\\ &+ 36K_2h_1h_2^227K_1h_1^2h_2^2 + 24K_2h_2^3 + 24K_1h_1h_2^3 + 15K_1h_2^4\\ &+ 84K_4(h_1 + h_2))),\end{aligned}$$

$$\begin{aligned}&R_1 = \frac{12}{5}BrL_1^2L_{1x}\lambda, \quad R_2 = \frac{24}{5}BrL_1L_{1x}L_2\lambda,\\ &R_3 = -\frac{1}{12}Br(72G_1L_1 - 24L_{1x}L_2^2\lambda - 24L_1L_2L_{2x}\lambda - 36L_1L_{1x}L_3\lambda\\ &+ 36L_1^2L_{3x}\lambda),\end{aligned}$$

$$\begin{aligned}R_4 = &-\frac{1}{6}Br(24G_2L_1 + 24G_1L_2 - 8L_2^2L_{2x}\lambda - 12L_{1x}L_2L_3\lambda - 12L_1L_{2x}L_3\lambda\\ &+ 12L_1L_2L_{3x}\lambda + 36L_1^2L_{4x}\lambda),\end{aligned}$$

$$R_5 = -\frac{1}{2}Br(8G_2L_2 - 4L_2L_{2x}L_3\lambda + 12L_1L_2L_{4x}\lambda),$$

$$\begin{aligned}R_6 = &\frac{1}{60}(-2h_1^5R_1 - h_1^4(2h_2R_1 + 3R_2) - h_1^3(2h_2^2R_1 + 3h_2R_2 + 5R_3)\\ &- h_1^2(2h_2^3R_1 + 3h_2^2R_2 + 5h_2R_3 + 10R_4) - h_1(2h_2^4R_1 + 3h_2^3R_2\\ &+ 5h_2^2R_3 + 10h_2R_4 + 30R_5) - h_2(2h_2^4R_1 + 3h_2^3R_2 + 5h_2^2R_3\\ &+ 10h_2R_4 + 30R_5)),\end{aligned}$$

$$\begin{aligned}R_7 = &\frac{1}{60}h_1h_2(2h_1^4R_1 + 2h_2^4R_1 + 3h_2^3R_2 + h_1^3(2h_2R_1 + 3R_2) + 5h_2^2R_3\\ &+ h_1^2(2h_2^2R_1 + 3h_2R_2 + 5R_3) + 10h_2R_4 + h_1(2h_2^3R_1 + 3h_2^2R_2\\ &+ 5h_2R_3 + 10R_4) + 30R_5),\end{aligned}$$

$$\begin{aligned}&S_1 = -SrScR_1, \quad S_2 = -SrScR_2, \quad S_3 = -SrScR_3,\\ &S_4 = -SrScR_4, \quad S_5 = -SrScR_5,\end{aligned}$$

$$\begin{aligned}S_6 = &-\frac{1}{h_2}(-h_2^6R_1ScSr - h_2^5R_2ScSr - h_2^4R_3ScSr - h_2^3R_4ScSr - h_2^2R_5ScSr\\ &- h_2R_6ScSr - R_7ScSr) + \frac{1}{h_2(-h_1 + h_2)}(-h_1^6h_2R_1ScSr\\ &+ h_1h_2^6R_1ScSr - h_1^5h_2R_2ScSr + h_1h_2^5R_2ScSr - h_1^4h_2R_3ScSr\\ &+ h_1h_2^4R_3ScSr - h_1^3h_2R_4ScSr + h_1h_2^3R_4ScSr - h_1^2h_2R_5ScSr\\ &+ h_1h_2^2R_5ScSr + h_1h_2^5R_2ScSr + h_1R_7ScSr - h_2R_7ScSr),\end{aligned}$$

$$\begin{aligned}S_7 = &-\frac{1}{(-h_1 + h_2)}(-h_1^6h_2R_1ScSr + h_1h_2^6R_1ScSr - h_1^5h_2R_2ScSr\\ &+ h_1h_2^5R_2ScSr - h_1^4h_2R_3ScSr - h_1^3h_2R_4ScSr + h_1h_2^3R_4ScSr\\ &- h_1^2h_2R_5ScSr + h_1h_2^2R_5ScSr + h_1R_7ScSr - h_1h_2^4R_3ScSr\\ &- h_2R_7ScSr),\end{aligned}$$

$$\begin{aligned}&K_{10} = (336K_1 + 216L_1L_{1xx}L_{1x}\lambda^2), \quad K_{11}\\ &= (210K_2 + 288L_1^2L_{1xx}\lambda^2 - 90L_1L_{1xx}L_{1x}\lambda^2 - 90L_1L_{1x}^2\lambda^2\\ &+ 72L_{1xx}L_{1x}L_2\lambda^2 + 72L_{1xx}L_{1x}L_{2xx}\lambda^2 + 216L_1L_{1xx}L_{2x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_{12} = &(120K_3 + 96L_1L_{1xx}L_2\lambda^2 - 48L_{1xx}L_{1x}L_2\lambda^2 - 24L_{1x}^2L_2\lambda^2\\ &+ 288L_1^2L_{2xx}\lambda^2 - 72L_1L_{1x}L_{2xx}\lambda^2 + 24L_{1x}L_2L_{2xx}\lambda^2 - 96L_1L_{1x}\lambda^2\\ &+ 72L_{1xx}L_2L_{2x}\lambda^2 + 72L_1L_{2xx}L_{2x}\lambda^2 + 216L_1L_{1xx}L_{3x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_{13} = &(60K_4 + 144G_{1x}L_1 + 3L_{1xx} + 144G_1L_{1x} - 36G_{1x}L_1\lambda\\ &- 252G_1L_{1x}\lambda + 648L_1^2L_{1xx}\lambda^2 + 144L_1L_2L_{2xx}\lambda^2 - 36L_{1x}L_2L_{2xx}\lambda^2\\ &- 24L_{1x}L_2L_{2x}\lambda^2 + 24L_2L_{2xx}\lambda^2 + 36L_1L_{2x}^2\lambda^2 - 18L_{1xx}L_{1x}L_3\lambda^2\\ &+ 216L_1^2L_{3xx}\lambda^2 - 54L_1L_{1x}L_{3xx}\lambda^2 - 216L_1L_{1x}L_{3x}\lambda^2 + 72L_{1xx}L_2L_{3x}\lambda^2\\ &- 24L_{1xx}L_2^2\lambda^2 + 72L_1L_{2xx}L_{3x}\lambda^2 + 216L_1L_{1xx}L_{4x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_{14} = {} & (24K_5 + 48G_{2x}L_{1x} + 48G_{1x}L_2 + 2L_{2xx} + 48G_1L_{1x} - 24L_{1xx}L_2^2\lambda^2 \\ & - 24G_{1x}L_2 + 288L_1L_{1xx}L_2\lambda^2 + 144L_1^2L_{2xx}\lambda^2 + 8L_2L_{2x}^2\lambda^2 \\ & - 24L_{1xx}L_2L_3\lambda^2 + 24L_1L_{2xx}L_3\lambda^2 - 12L_{1x}L_{2xx}\lambda^2 + 120L_1L_2L_{3xx}\lambda^2 \\ & - 24L_{1x}L_2L_{3xx}\lambda^2 - 48L_{1x}L_2L_{3x}\lambda^2 + 24L_2L_{2xx}L_{3x}\lambda^2 + 24L_1L_{2x}L_{3x}\lambda^2 \\ & - 24L_{1xx}L_2^2\lambda^2 - 96G_1L_{2x}\lambda + 72L_1^2L_{4xx}\lambda^2 - 36L_1L_{1x}L_{4xx}\lambda^2 \\ & - 360L_1L_{1x}L_{4x}\lambda^2 + 72L_{1xx}L_2L_{4x}\lambda^2 + 72L_1L_{2xx}L_{4x}\lambda^2),\end{aligned}$$

$$\begin{aligned}K_{15} = {} & (6K_6 + 16G_{2x}L_2 + 16G_2L_{2x} + L_{3xx} + 6G_{3x}L_1\lambda - 6G_3L_{1x}\lambda \\ & - 4G_{2x}L_2\lambda - 20G_2L_{2x}\lambda - 6G_{1x}L_3\lambda - 18G_1L_{3x}\lambda + 24L_{1xx}L_2^2\lambda^2 \\ & + 48L_1L_2L_{2xx}\lambda^2 - 8L_2L_{2xx}L_3\lambda^2 - 4L_{2x}^2L_3\lambda^2 + 24L_1L_3L_{3xx}\lambda^2 \\ & - 6L_{1x}L_3L_{3xx}\lambda^2 + 8L_2L_{3x}\lambda^2 + 12L_{1x}L_3L_{3x}\lambda^2 + 6L_1L_{3x}^2\lambda^2 \\ & + 24L_1L_2L_{4xx}\lambda^2 + 8L_2^3L_{3xx}\lambda^2 - 12L_{1x}L_2L_{4xx}\lambda^2 - 96L_{1x}L_2L_{4x}\lambda^2 \\ & + 24L_2L_{2xx}L_{4x}\lambda^2 - 24L_1L_{2x}L_{4x}\lambda^2),\end{aligned}$$

$$C_1 = -\frac{1}{56}(36BrL_1(56K_1 + 3L_1(8L_1L_{1xx} + L_{1x}(-L_{1xx} + 5L_{1x}))\lambda^2)),$$

$$\begin{aligned}C_2 = {} & -\frac{1}{42}(12Br(105K_2L_1 + 56K_1L_2 + 3L_1(L_{1x}(-3L_{1xx} + 7L_{1x})L_2 \\ & + 12L_1^2L_{2xx} + L_1(-18L_{1x}^2 + 36L_{1xx}L_2 - 3L_{1x}L_{2xx} + 23L_{1x}L_{2x}))\lambda^2)),\end{aligned}$$

$$\begin{aligned}C_3 = {} & \frac{1}{30}(12Br(60K_3L_1 + 3L_{1x}^2 + 35K_2L_2 + 108L_1^3L_{1xx}\lambda^2 - 2L_{1xx}L_{1x}L_2^2\lambda^2 \\ & + 2L_{1x}^2L_2^2\lambda^2 + 3L_1^2\lambda(-2G_{1x} + 20L_2L_{2xx} + 2(7L_{2x}^2 + 6L_{1xx}L_3) \\ & - 3L_{1x}(4L_{2x} + L_{3xx}))\lambda) - L_1(L_{1x}\lambda(42G_1 + L_2(54L_{1x} + 9L_{2xx} \\ & - 31L_{2x})\lambda) + L_{1xx}(1 - 48L_2^2\lambda^2 + 3L_{1x}L_3\lambda^2))),\end{aligned}$$

$$\begin{aligned}C_4 = {} & \frac{1}{20}(4Br(90K_4L_1 + 60K_3L_2 - L_{1xx}L_2 - 3L_1L_{2xx} + 12L_{1x}L_{2x} \\ & - 54G_2L_1L_{1x}\lambda - 24G_{1x}L_1L_2\lambda - 42G_1L_{1x}L_2\lambda - 72G_1L_1L_{2x}\lambda \\ & + 324L_1^2L_{1xx}L_2\lambda^2 - 36L_{1x}^2L_2^2\lambda^2 + 20L_{1xx}L_2^3\lambda^2 + 108L_1^3L_{2xx}\lambda^2 \\ & + 84L_1L_2^2L_{2xx}\lambda^2 - 6L_{1x}L_2^2L_{2xx}\lambda^2 - 108L_1L_{1x}L_2L_{2x}\lambda^2 + 8L_{1x}L_2^2L_{2x}\lambda^2 \\ & - 54L_1L_{1x}^2L_3\lambda^2 + 72L_1L_{1xx}L_2L_3\lambda^2 - 3L_{1xx}L_{1x}L_2L_3\lambda^2 \\ & + 72L_1^2L_{2xx}L_3\lambda^2 - 9L_1L_{1x}L_{2xx}L_3\lambda^2 + 36L_1^2L_2L_{3xx}\lambda^2 \\ & - 27L_1L_{1x}L_2L_{3xx}\lambda^2 - 54L_1^2L_{1x}L_{3x}\lambda^2 - 54L_1L_{1x}L_2\lambda^2 \\ & + 144L_1^2L_{2x}L_{3x}\lambda^2 - 108L_1^3L_{4xx}\lambda^2 - 27L_1^2L_{1x}L_{4xx}\lambda^2 \\ & - 270L_1^2L_{1x}L_{4x}\lambda^2)),\end{aligned}$$

$$\begin{aligned}C_5 = {} & \frac{1}{12}(4Br(36K_5L_1 + 30K_4L_2 - L_2L_{2xx} + 4L_{2x}^2 - 3L_1L_{3xx} + 6L_{1x}L_{3x} + 9G_{3x}L_1^2\lambda \\ & - 9G_3L_1L_{1x}\lambda - 6G_{2x}L_1L_2\lambda - 18G_2L_{1x}L_2\lambda - 6G_{1x}L_2^2\lambda - 30G_2L_1L_{2x}\lambda \\ & - 24G_1L_2L_{2x}\lambda - 9G_{1x}L_1L_3\lambda - 27G_1L_1L_{3x}\lambda + 108L_1L_{1xx}L_2^2\lambda^2 \\ & + 108L_1^2L_2L_{2xx}\lambda^2 + 12L_2^3L_{2xx}\lambda^2 + 72L_1L_2L_{2x}L_{3x}\lambda^2 - 24L_{1x}L_2^2L_{2x}\lambda^2 \\ & + 10L_2^2L_{2x}^2\lambda^2 - 18L_{1x}^2L_2L_3\lambda^2 + 12L_{1xx}L_2^2L_3\lambda^2 + 48L_1L_2L_{2xx}L_3\lambda^2 \\ & - 3L_{1x}L_2L_{2xx}L_3\lambda^2 - 36L_1L_{1x}L_{2x}L_3\lambda^2 + 6L_1L_{2x}^2L_3\lambda^2 + 24L_1L_2^2L_{3xx}\lambda^2 \\ & - 18L_1L_{1x}L_3L_{3x}\lambda^2 - 6L_{1x}L_2^2L_{3xx}\lambda^2 + 36L_1^2L_3L_{3xx}\lambda^2 - 9L_1L_{1x}L_3L_{3xx}\lambda^2 \\ & - 54L_1L_{1x}L_2L_{3x}\lambda^2 - 18L_{1x}L_2^2L_{3x}\lambda^2 + 63L_1^2L_{3x}^2\lambda^2 - 108L_1^2L_2L_{4xx}\lambda^2 \\ & - 27L_1L_{1x}L_2L_{4xx}\lambda^2 - 234L_1L_{1x}L_2L_{4x}\lambda^2 - 36L_1^2L_{2x}L_{4x}\lambda^2)),\end{aligned}$$

$$\begin{aligned}C_6 = {} & \frac{1}{6}(4Br(9K_6L_1 + 12K_5L_2 - L_2L_{3xx} + 4L_{2x}L_{3x} - 3L_1L_{4xx} + 9G_{4x}L_1^2\lambda \\ & + 3G_{3x}L_1L_2\lambda - 3G_{3x}L_1L_2\lambda - 2G_{2x}L_2^2\lambda - 3G_3L_1L_{2x}\lambda - 10G_2L_2L_{2x}\lambda \\ & - 3G_{2x}L_1L_3\lambda - 3G_{1x}L_2L_3\lambda - 12G_2L_1L_{3x}\lambda - 9G_1L_2L_{3x}\lambda \\ & + 9G_1L_1L_{4x}\lambda + 12L_{1xx}L_2^3L_2^3\lambda^2 + 36L_1L_2^2L_{2xx}\lambda^2 + 8L_2^2L_{2xx}\lambda^2 \\ & - 12L_{1x}L_2L_{2x}L_3\lambda^2 + 2L_2L_{2x}^2L_3\lambda^2 + 4L_2^3L_{3xx}\lambda^2 + 24L_1L_2L_3L_{3xx}\lambda^2 \\ & - 3L_{1x}L_2L_3L_{3xx}\lambda^2 - 12L_{1x}L_2^2L_{3x}\lambda^2 + 8L_2^2L_{2x}L_{3x}\lambda^2 - 18L_1L_{1x}L_3L_{3x}\lambda^2 \\ & - 9L_1L_{1x}L_3L_{4xx}\lambda^2 - 6L_{1x}L_2L_3L_{3x}\lambda^2 + 3L_1L_{2x}L_3L_{3x}\lambda^2 + 33L_1L_2L_{3x}^2\lambda^2 \\ & - 36L_1L_2^2L_{4xx}\lambda^2 - 6L_{1x}L_2^2L_{4xx}\lambda^2 - 48L_{1x}L_2^2L_{4x}\lambda^2 - 42L_1L_2L_{2x}\lambda^2 \\ & - 27L_1L_{1x}L_3L_{4x}\lambda^2 + 27L_1^2L_{3x}L_{4x}\lambda^2)),\end{aligned}$$

$$\begin{aligned}C_7 = {} & \frac{1}{2}(4Br(3K_6L_2 + L_2L_{3x}\lambda(-4G_2 + (-6L_{1x}L_3 + L_{2x}L_3 + 9L_1L_{4x})\lambda) \\ & + L_{3x}^2(1 + 4L_2^2\lambda^2) - L_2(\lambda(-3G_{4x}L_1 + G_3L_{2x} + G_{2x}L_3 - 3G_1L_{4x} \\ & - 4L_2^2L_{2xx}\lambda - 4L_2^2L_{2xx}\lambda - 4L_2L_3L_{3xx}\lambda + 10L_2L_{2x}L_{4x}\lambda + 9L_{1x}L_3L_{4x}\lambda) \\ & + L_{4xx}(1 + 4L_2^2\lambda^2 + 3L_{1x}L_3\lambda^2)))),\end{aligned}$$

$$\begin{aligned}C_8 = {} & \frac{1}{840}(\frac{1}{(h_2 - h_1)}(h_1(70C_5h_1^3 + 42C_4h_1^4 + 28C_3h_1^5 + 20C_2h_1^6 \\ & + 15C_1h_1^7 + 420C_7(h_1 - h_2) - 70C_5h_2^3 - 42C_4h_2^4 - 28C_3h_2^5 \\ & - 20C_2h_2^6 - 15C_1h_2^7 + 140C_6(h_1^2 - h_2^2))) - h_2(420C_7 \\ & + h_2(140C_6 + h_2(70C_5 + 42C_4h_2 + 28C_3h_2^2 + 20C_2h_2^3 \\ & + 15C_1h_2^4)))),\end{aligned}$$

$$\begin{aligned}C_9 = {} & \frac{1}{840(h_1 - h_2)}(h_1h_2(70C_5h_1^3 + 42C_4h_1^4 + 28C_3h_1^5 + 20C_2h_1^6 \\ & + 15C_1h_1^7 + 420C_7(h1 - h2) - 70C_5h_2^3 - 42C_4h_2^4 - 28C_3h_2^5 \\ & - 20C_2h_2^6 - 15C_1h_2^7 + 140C_6(h_1^2 - h_2^2)),\end{aligned}$$

$$C_{10} = C_1ScSr, \quad C_{11} = C_2ScSr, \quad C_{12} = \frac{1}{20}(B_{1xx} + P_{1xx}ScSr) + C_3ScSr,$$

$$\begin{aligned}C_{13} & = \frac{1}{12}(B_{2xx} + P_{2xx}ScSr) + C_4ScSr, \\ C_{14} & = \frac{1}{6}(B_{3xx} + P_{3xx}ScSr) + C_5ScSr, \\ C_{15} & = \frac{1}{2}(B_{4xx} + P_{4xx}ScSr) + C_6ScSr, \\ C_{16} & = (B_{5xx} + P_{5xx}ScSr) + C_7ScSr,\end{aligned}$$

$$\begin{aligned}C_{17} = {} & \frac{1}{840}(\frac{1}{(h_2 - h_1)}(h_1(70C_1^4h_1^3 + 42C_{13}h_1^4 + 28C_{12}h_1^5 + 20C_{11}h_1^6 \\ & + 15C_{10}h_1^7 + 420C_{16}(h_1 - h_2) - 70C_{14}h_2^3 - 42C_{13}h_2^4 \\ & - 28C_{12}h_2^5 - 20C_{11}h_2^6 - 15C_{10}h_2^7 + 140C_{15}(h_1^2 - h_2^2))) \\ & - h2(420C_{16} + h2(140C_{15} + h2(70C_{14} + 42C_{13}h_2 + 28C_{12}h_2^2 \\ & + 20C_{11}h_2^3 + 15C_{10}h_2^4)))),\end{aligned}$$

$$\begin{aligned}C_{18} = {} & \frac{1}{840(h_1 - h_2)}(h_1h_2(70C_{14}h_1^3 + 42C_{13}h_1^4 + 28C_{12}h_1^5 + 20C_{11}h_1^6 \\ & + 15C_{10}h_1^7 + 420C_{16}(h_1 - h_2) - 70C_{14}h_2^3 - 42C_{13}h_2^4 \\ & - 28C_{12}h_2^5 - 20C_{11}h_2^6 - 15C_{10}h_2^7 + 140C_{15}(h_1^2 - h_2^2)),\end{aligned}$$

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