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ORIGINAL ARTICLE

Possible formation of compact stars in $f(R,T)$ gravity

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Abstract This paper reports on the investigations regarding the possible formation of compact stars in $f(R,T)$ theory of gravity, where R is the Ricci scalar and T is the trace of the energy–momentum tensor. In this connection, we use the analytic solution of the Krori and Barua metric (Krori and Barua in J. Phys. A., Math. Gen. 8:508, 1975) for a spherically symmetric anisotropic star in the context of $f(R,T)$ gravity. The masses and radii of compact star models, namely Model 1, Model 2, and Model 3, are employed to incorporate the unknown constants in the Krori and Barua metric. The physical features such as regularity at the center, the anisotropy measure, causality, and the well-behaved condition of the above-mentioned class of compact starts are analyzed. Moreover, we also discuss the energy conditions, stability, and surface redshift in $f(R,T)$ gravity.

Keywords Compact stars · $f(R,T)$ gravity

1 Introduction

The late time evolution of stars influenced by a strong gravitational pull has been a largely anticipated field in astrophysics and gravitational theories. It facilitates investigations of diverse characteristics of gravitating sources via various physical phenomena. Baade and Zwicky (1934) proposed the inception of massive compact stellar objects, establishing the argument that a supernova may result in a small and super dense star. This eventually came to reality in 1967, when Bell and Hewish (Longair 1994; Ghosh 2007) discovered pulsars that are highly magnetized and rotating neutron stars. So, in reality, we come across a fundamental revolutionary shift from normal stars to compact stars, with a wide range in the form of stars to neutron stars, quarks, dark stars, gravastars, and finally black holes.

In our present work, we are more specifically interested in the study of the compact-star category. Generally the homogeneity of the spherically symmetric matter configuration is emphasized, while theoretical modeling of a compact star, satisfying the Tolman–Oppenheimer–Volkoff (TOV) equation. Ruderman (1972) was first to argue that the nuclear matter density becomes anisotropic at the core of a compact object. Analytic solutions of the field equations for various static spherically symmetric configurations with anisotropy compatibility with the interior of compact stellar modeling have been obtained in numerous works (Maurya and Gupta 2012, 2013, 2014; Maharaj et al. 2014; Pant et al. 2014a, 2014b). The pressure inside the fluid sphere is split in two parts, namely the radial and tangential pressure. It has been investigated that anisotropy gives rise to the repulsive force that assists to maintain the compact objects. In this context, it is established in Kalam et al. (2012) that the Krori and Barua (henceforth KB) (1975) metric provides an effective and realistic approach in the modeling of compact stars.

The numerical simulations can be taken into account to explore the characteristics of compact stars from the integrated TOV equations, if the equation of state (EoS) is known. Rahaman et al. (2012a, 2012b) extended the KB

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models by using the Chaplygin gas EoS and discussed their physical features. Mak and Harko (2004) established standard models of spherically symmetric compact stars via exact solution of the field equations. They determined the physical parameters such as the energy density and the tangential and radial pressure, concluding that these parameters remain finite and positive inside the stars. The anisotropic exact models for compact objects with a barotropic EoS are discussed in Lobo (2006). Hossein et al. (2012) studied the impact of the cosmological constant on anisotropic compact stars.

General Relativity (GR), a fundamental theory of gravity, is successful in the weak field limit, but insufficient to explore strong fields. The expected description of GR in the strong field regime can be done by its modifications. The modifications in the Einstein–Hilbert (EH) action are induced to arrive at alternative theories of gravity. Among the modified theories of gravity, $f(R)$ gravity, being the most elementary modification of GR, is extensively studied in the context of the existence and stability of neutron stars and compact stars (Arapoglu et al. 2011; Alavirad and Weller 2013; Astashenok et al. 2014, 2015; Yazadjiev et al. 2014; Kausar and Noureen 2014; Noureen et al. 2015). Abbas et al. (2014, 2015a, 2015b, 2015c) analyzed a class of compact stars in GR, $f(T)$, where T is the torsion scalar, with a different equation of state.

The issue of the accelerated expansion of the universe can be explained by taking into account the modified theories of gravity such as $f(R,T)$ gravity (Harko et al. 2011). $f(R,T)$ gravity provides an alternative way to explain the current cosmic acceleration without the need of introducing either an extra spatial dimension or an exotic component of dark energy. Harko et al. (2011) generalized $f(R)$ gravity by introducing an arbitrary function of the Ricci scalar R and the trace of the energy–momentum tensor T. The dependence of T may be introduced by exotic imperfect fluids or quantum effects (conformal anomaly). As a result of the coupling between matter and geometry, the motion of test particles is non-geodesic and an extra acceleration is always present. In $f(R,T)$ gravity, cosmic acceleration may result not only due to a geometrical contribution to the total cosmic energy density, but it also depends on the matter contents.

Soon after the genesis of $f(R,T)$ gravity, its cosmological and thermodynamic implications, including the energy conditions and dynamical analysis, were extensively discussed (Shabani and Farhoudi 2014; Harko and Lobo 2010; Harko and Lobo 2010; Azizi 2013; Sharif and Zubair 2012a, 2012b, 2013; Jamil et al. 2012a, 2012b; Momeni et al. 2015a, 2015b; Momeni and Myrzakulov 2015; Barrientos and Rubilar 2014). However, the explorations regarding compact stars in $f(R,T)$ gravity are yet to be done. Herein, we are interested in the study of the structure of a class of compact stars, Model 1, Model 2, and Model 3 in $f(R,T)$ gravity.

The modified action in $f(R,T)$ is as follows (Harko et al. 2011):

$$\int dx^4\sqrt{-g}\left[\frac{f(R,T)}{16\pi G}+\mathcal{L}_{(m)}\right], \tag{1.1}$$

where $\mathcal{L}_{(m)}$ is the matter Lagrangian and g denotes the metric tensor. Different choices of $\mathcal{L}_{(m)}$ can be considered, each of which leads to a specific form of fluid. In our present work the viable $f(R,T)$ model we have chosen is of the following type:

$$f(R,T)=f_1(R)+f_2(T), \tag{1.2}$$

where $f_1(R)$ is a function of the Ricci scalar and λ is some positive constant value. We have analyzed the above-mentioned $f(R,T)$ model with $f_1(R)=R+\alpha R^2$, α being a positive scalar. In our discussion, we shall consider the model $f(R,T)=f_1(R)+f_2(T)$, which does not imply the direct non-minimal gravitational coupling between the scalar curvature R and the trace of the energy–momentum tensor T in the Lagrangian level. But there may be a coupling between matter and geometry which becomes apparent in the study of thermodynamics (Sharif and Zubair 2012a). The $f_2(T)$ can be considered as a matter correction term to $f(R)$ gravity and in this particular study we choose $f_2(T)=\lambda T$. The main reason behind the difference as regards cosmology in ordinary $f(R)$ gravity and in the above $f(R,T)$ model is the non-trivial coupling between matter and geometry. In our previous work (Sharif and Zubair 2012b), we have reconstructed some explicit models of this $f(R,T)$ gravity for an anisotropic universe and explored the phantom era of dark energy.

We have investigated that only for $\lambda=1$ and $\alpha=2$ the energy density of all the considered models remains positive and the energy conditions are only valid for these values of λ and α. So, throughout the analysis, we have used the above values of λ and α, and this will not be mentioned again explicitly.

This paper is arranged as follows: In the following section the anisotropic matter distribution and the expressions of the physical parameters for the energy density ρ, and the radial and tangential pressure are established. Section 3 constitutes the analysis of physical features of compact stars and their stability analysis. In the last section, we summarize our results.

2 Anisotropic matter distribution in $f(R,T)$ gravity

The line element for our particular spherically symmetric metric describing the compact star stellar configuration is

$$ds^2=e^{a(r)}dt^2-e^{b(r)}dr^2-r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \tag{2.1}$$

where $b=Ar^2$, $a=Br^2+C$ (Krori and Barua 1975), A, B, and C are arbitrary constants, which will be calculated by using some physical assumptions. The above set of functions are introduced to arrive at a singularity-free structure for compact stars. Clearly, this set of functions leads to a non-singular density and curvature setting.

Taking $8\pi G=1$ and upon variation of the modified EH action in $f(R,T)$ (1.1) with respect to the metric tensor g_{uv}, the following modified field equations are formed:

$$\begin{aligned} G_{uv}={}&\frac{1}{f_R}\biggl[(f_T+1)T^{(m)}_{uv}-\rho g_{uv}f_T+\frac{f-Rf_R}{2}g_{uv}\\ &+(\nabla_u\nabla_v-g_{uv}\Box)f_R\biggr], \end{aligned} \tag{2.2}$$

where $f_R=\frac{\partial f(R,T)}{\partial R}$, $f_T=\frac{\partial f(R,T)}{\partial T}$, and $T^{(m)}_{uv}$ denotes the usual matter energy–momentum tensor, which is considered to be anisotropic and is given by

$$T^{(m)}_{uv}=(\rho+p_t)V_uV_v-p_tg_{uv}+(p_r-p_t)\chi_u\chi_v, \tag{2.3}$$

where ρ, p_r, and p_t denote the energy density and the radial and transverse stresses, respectively. The four-velocity is denoted by V_u and χ_u is a radial four-vector; they satisfy

$$\begin{aligned} &V^u=e^{\frac{-a}{2}}\delta^u_0, \qquad V^uV_u=1,\\ &\chi^u=e^{\frac{-b}{2}}\delta^u_1, \qquad \chi^u\chi_u=-1. \end{aligned} \tag{2.4}$$

When $f(R,T)=f_1(R)+\lambda T$, the expression for ρ, p_r, and p_t can be extracted from the modified field equations as follows:

$$\begin{aligned} \rho={}&\frac{1}{2(1+2\lambda)}\biggl[\frac{2+5\lambda}{e^b(1+\lambda)}\biggl\{\biggl(\frac{a'}{r}-\frac{a'b'}{4}+\frac{a''}{2}+\frac{a'^2}{4}\biggr)f_{1R}\\ &-f''_{1R}+\biggl(\frac{b'}{2}-\frac{f_1}{2}e^b-\frac{2}{r}\biggr)f'_{1R}\biggr\}+\frac{\lambda}{e^b(1+\lambda)}\\ &\times\biggl\{\biggl(\frac{a'b'}{4}+\frac{b'}{r}-\frac{a''}{2}-\frac{a'^2}{4}\biggr)f_{1R}+\biggl(\frac{a'}{2}+\frac{2}{r}\biggr)f'_{1R}\\ &+\frac{f_1}{2}e^b\biggr\}+\frac{2\lambda}{e^b(1+\lambda)}\biggl\{\frac{f_{1R}}{r^2}\biggl(\frac{(b'-a')r}{2}-e^b+1\biggr)\\ &+\biggl(\frac{a'-b'}{2}+\frac{1}{r}\biggr)f'_{1R}+f''_{1R}+\frac{f_1}{2}e^b\biggr\}\biggr], \end{aligned} \tag{2.5}$$

$$\begin{aligned} p_r={}&\frac{-1}{2(1+2\lambda)}\biggl[\frac{\lambda}{e^b(1+\lambda)}\biggl\{\biggl(\frac{a'}{r}-\frac{a'b'}{4}+\frac{a''}{2}+\frac{a'^2}{4}\biggr)f_{1R}\\ &-f''_{1R}+\biggl(\frac{b'}{2}-\frac{f_1}{2}e^b-\frac{2}{r}\biggr)f'_{1R}\biggr\}\\ &-\frac{(2+3\lambda)}{e^b(1+\lambda)}\biggl\{\biggl(\frac{a'b'}{4}+\frac{b'}{r}-\frac{a''}{2}-\frac{a'^2}{4}\biggr)f_{1R}\\ &+\biggl(\frac{a'}{2}+\frac{2}{r}\biggr)f'_{1R}+\frac{f_1}{2}e^b\biggr\}+\frac{2\lambda}{e^b(1+\lambda)}\\ &\times\biggl\{\frac{f_{1R}}{r^2}\biggl(\frac{(b'-a')r}{2}-e^b+1\biggr)\\ &+\biggl(\frac{a'-b'}{2}+\frac{1}{r}\biggr)f'_{1R}+f''_{1R}+\frac{f_1}{2}e^b\biggr\}\biggr], \end{aligned} \tag{2.6}$$

$$\begin{aligned} p_t={}&\frac{-1}{2(1+2\lambda)}\biggl[\frac{\lambda}{e^b(1+\lambda)}\biggl\{\biggl(\frac{a'}{r}-\frac{a'b'}{4}+\frac{a''}{2}+\frac{a'^2}{4}\biggr)f_{1R}\\ &-f''_{1R}+\biggl(\frac{b'}{2}-\frac{f_1}{2}e^b-\frac{2}{r}\biggr)f'_{1R}\biggr\}\\ &-\frac{\lambda}{e^b(1+\lambda)}\biggl\{\biggl(\frac{a'b'}{4}+\frac{b'}{r}-\frac{a''}{2}-\frac{a'^2}{4}\biggr)f_{1R}\\ &+\biggl(\frac{a'}{2}+\frac{2}{r}\biggr)f'_{1R}+\frac{f_1}{2}e^b\biggr\}\\ &-\frac{2}{e^b}\biggl\{\frac{f_{1R}}{r^2}\biggl(\frac{(b'-a')r}{2}-e^b+1\biggr)\\ &+\biggl(\frac{a'-b'}{2}+\frac{1}{r}\biggr)f'_{1R}+f''_{1R}+\frac{f_1}{2}e^b\biggr\}\biggr]. \end{aligned} \tag{2.7}$$

Here $f_{1R}=\frac{df_1}{dR}$ and a prime denotes the derivative with respect to the radial coordinate. Substituting Eq. (1.2) in Eqs. (2.5)–(2.7) and inserting the value of the Ricci scalar in the form of metric functions together with the KB metric coefficients, we arrive at

$$\begin{aligned} \rho={}&\frac{1}{r^4(1+3\lambda+2\lambda^2)}\bigl[e^{-2Ar^2}\bigl\{e^{2Ar^2}\bigl(r^2+2\alpha(\lambda-1)\bigr)\\ &+2\alpha\bigl(-1-3Br^2-B^2r^4+Ar^2\bigl(2+Br^2\bigr)\bigr)\\ &\times\bigl(1-\lambda-3Br^2(1+3\lambda)+B^2r^4(3+7\lambda)\bigr)\\ &-e^{Ar^2}\bigl(4\alpha(\lambda-1)+r^2(1+24B\alpha\lambda)+8A\alpha(1+\lambda)\\ &+Br^6\bigl(A\lambda+B(2+4\lambda)\bigr)\bigr)-r^4\bigl(A\bigl(2+(4+8B\alpha)\lambda\bigr)\\ &+B\bigl(3\lambda+4B\alpha(2+3\lambda)\bigr)\bigr)\bigr\}\bigr], \end{aligned} \tag{2.8}$$

$$\begin{aligned} p_r={}&\frac{1}{r^4(1+3\lambda+2\lambda^2)}\bigl[e^{-2Ar^2}\bigl\{e^{2Ar^2}(r^2+2\alpha(\lambda-1)\\ &+e^{Ar^2}\bigl(4\alpha(\lambda-1)+ABr^6\lambda+r^2\bigl(1+8B\alpha(\lambda-1)\\ &-8A\alpha\lambda\bigr)+Br^4\bigl(2+\lambda-4\alpha\lambda(2A-B)\bigr)\bigr)\\ &-2\alpha\bigl(1+3Br^2+B^3r^4-Ar^2\bigl(2+Br^2\bigr)\bigr)\\ &\times\bigl(-1+Br^2(\lambda-1)-Ar^2\bigl(2(1+\lambda)\\ &+Br^2(1+3\lambda)\bigr)\bigr)\bigr\}\bigr], \end{aligned} \tag{2.9}$$

$$\begin{aligned} p_t={}&\frac{1}{r^4(1+3\lambda+2\lambda^2)}\bigl[e^{-2Ar^2}\bigl\{2\alpha\bigl(1+3Br^2+B^2r^4\\ &-Ar^2\bigl(2+Br^2\bigr)\bigr)\bigl(-3-B^2r^4-2\lambda+Br^2\bigl(Ar^2-1\bigr)\\ &\times(1+2\lambda)\bigr)+e^{2Ar^2}\bigl(r^2(2+\lambda)-2\alpha(3+2\lambda)\bigr) \end{aligned}$$

Table 1 Approximate values of the model parameters for the compact stars considered

Models	M	R (km)	$u=\frac{M}{R}$	A (km^{-2})	B (km^{-2})	Z_s
Model 1	$0.88M_\odot$	7.7	0.168	0.006906276428	0.004267364618	0.23
Model 2	$1.435M_\odot$	7.07	0.299	0.01823156974	0.01488011569	0.57
Model 3	$2.25M_\odot$	10.0	0.332	0.01090644119	0.009880952381	0.073

$$\begin{aligned}&+e^{Ar^2}(4\alpha(3+2\lambda)+Br^6(A-B+A\lambda)\\&-r^2(2+\lambda+4A\alpha(3+2\lambda)-4B\alpha(5+4\lambda))\\&+r^4(B(4B\alpha-1)(2+\lambda)\\&+A(1-8B\alpha(1+\lambda))))\}].\end{aligned}\tag{2.10}$$

The Schwarzschild solution is considered to be most suitable choice for matching the conditions in the exterior regime (Noureen and Zubair 2015, 2014; Goswami et al. 2014; Cooney et al. 2010; Ganguly et al. 2014). The interior metric of the boundary surface will be the same for the internal or external geometry of the star. Here, the exterior metric of the star is described by the Schwarzschild solution, given by

$$\begin{aligned}ds^2=&\left(1-\frac{2M}{r}\right)dt^2-\left(1-\frac{2M}{r}\right)^{-1}dr^2\\&-r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right).\end{aligned}\tag{2.11}$$

Smooth matching of the interior metric (2.1) to the vacuum exterior solution at the boundary surface, $r=R$, yields

$$g_{tt}^-=g_{tt}^+,\qquad g_{rr}^-=g_{rr}^+,\qquad \frac{\partial g_{tt}^-}{\partial r}=\frac{\partial g_{tt}^+}{\partial r},\tag{2.12}$$

where the superscripts $-$ and $+$ stand for the interior and exterior solutions. Matching of the interior and exterior spacetimes leads to the following:

$$A=-\frac{1}{R^2}\ln\left(1-\frac{2M}{R}\right),\tag{2.13}$$

$$B=\frac{M}{R^3}\left(1-\frac{2M}{R}\right)^{-1},\tag{2.14}$$

$$C=\ln\left(1-\frac{2M}{R}\right)-\frac{M}{R}\left(1-\frac{2M}{R}\right)^{-1}.\tag{2.15}$$

The values of the constants A and B are evaluated by using approximate values of M and R (Lattimer and Steiner 2014; Li et al. 1999) of the considered compact stars, provided in Table 1. The compactness of a star can be defined by $u=\frac{M(R)}{R}$, and the surface redshift Z_s can be determined by using the result $Z_s=(1-2u)^{-1/2}-1$. The values of Z_s for the considered models are given in Table 1.

3 Physical analysis

This section covers the physical constraints required for the interior solution, incorporating the anisotropic behavior, matching, and energy conditions together with the stability analysis of the considered compact stars.

3.1 Anisotropic behavior

Prior to the discussion of anisotropy measure, we discuss the evolution of the energy density ρ and the anisotropic stresses p_r and p_t, respectively, shown in Figs. 1–3.

The radial derivative of Eq. (2.5) leads to the following expression:

$$\begin{aligned}\frac{d\rho}{dr}=&\frac{-4e^{-2Ar^2}}{r^5(1+\lambda)(1+2\lambda)}(e^{2Ar^2}(r^2+2\alpha(-1+\lambda))\\&+2(-1-3Br^2-B^2r^4+Ar^2(2+Br^2)))\\&\times\alpha(1-\lambda-3Br^2(1+3\lambda)\\&+Ar^2(-2+Br^2)(1+3\lambda)\\&+B^2r^4(3+7\lambda))-e^{Ar^2}(4\alpha(-1+\lambda)\\&+r^2(1+24B\alpha\lambda+8A\alpha(1+\lambda))\\&+Br^6(A\lambda+B(2+4\lambda))-r^4(A(2+(4+8B\alpha)\lambda)\\&+B(3\lambda+4B\alpha(2+3\lambda))))))-\frac{4Ae^{-2Ar^2}}{r^3(1+\lambda)(1+2\lambda)}\\&\times(e^{2Ar^2}(r^2+2\alpha(-1+\lambda))+2(-1-3Br^2-B^2r^4\\&+Ar^2(2+Br^2))\alpha(1-\lambda-3Br^2(1+3\lambda)\\&+Ar^2(-2+Br^2)(1+3\lambda)+B^2r^4(3+7\lambda))\\&-e^{Ar^2}(4\alpha(-1+\lambda)+r^2(1+24B\alpha\lambda\\&+8A\alpha(1+\lambda))+Br^6(A\lambda+B(2+4\lambda))\\&-r^4(A(2+(4+8B\alpha)\lambda)\\&+B(3\lambda+4B\alpha(2+3\lambda))))))-\frac{e^{-2Ar^2}}{r^4(1+\lambda)(1+2\lambda)}\\&\times(2e^{2Ar^2}r+4Ae^{2Ar^2}r(r^2+2\alpha(-1+\lambda))\end{aligned}$$

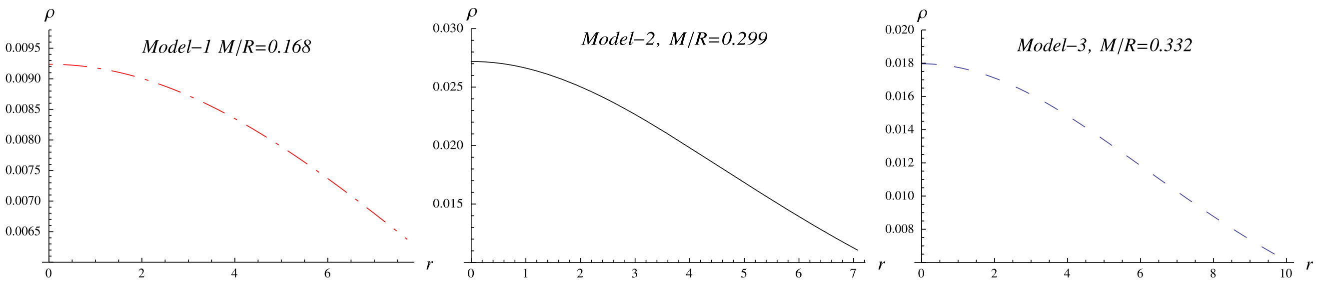

Fig. 1 Evolution of the energy density ρ with radius r

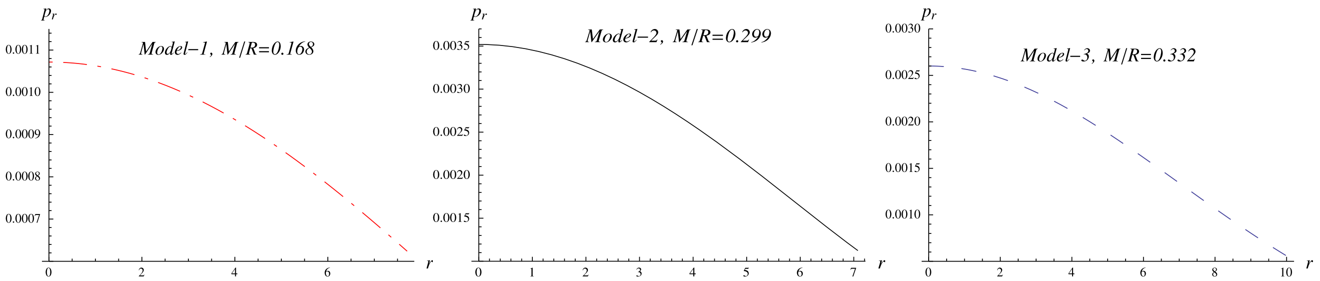

Fig. 2 Variation of the radial pressure p_r with radius r

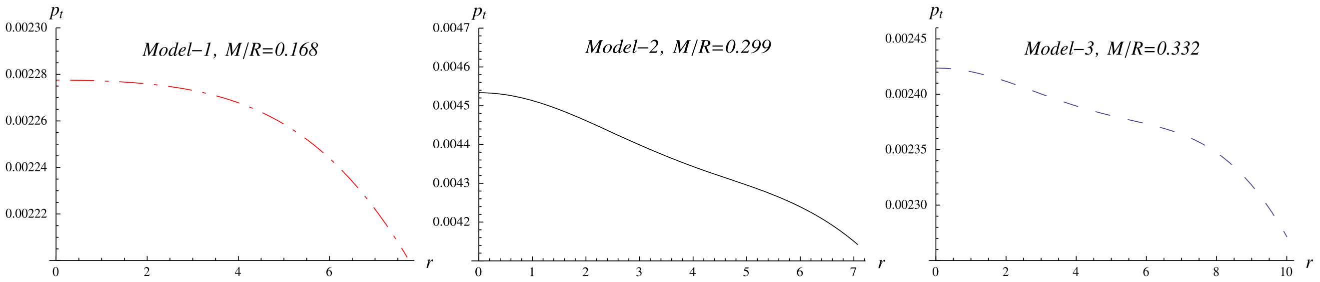

Fig. 3 Evolution of the tangential pressure p_r with radius r

$$
\begin{aligned}
&+2\big(-1-3Br^2-B^2r^4+Ar^2\big(2+Br^2\big)\big)\\
&\times\alpha\big(-6Br(1+3\lambda)+2ABr^3(1+3\lambda)\\
&+2Ar\big(-2+Br^2\big)(1+3\lambda)+(3+7\lambda)4B^2r^3\big)\\
&+2\big(-6Br+2ABr^3-4B^2r^3+2Ar\big(2+Br^2\big)\big)\\
&\times\lambda\big(1-\lambda-3Br^2(1+3\lambda)+Ar^2\big(-2+Br^2\big)\\
&\times(1+3\lambda)+B^2r^4(3+7\lambda)\big)-e^{Ar^2}\big(2r\big(1+24B\alpha\lambda\\
&+(1+\lambda)8A\lambda\big)+6Br^5\big(A\lambda+B(2+4\lambda)\big)\\
&-4r^3\big(A\big(2+(4+8B\alpha)\lambda\big)\\
&+B\big(3\lambda+(2+3\lambda)4B\alpha\big)\big)\big)-2Ae^{Ar^2}r\big(4\alpha(-1+\lambda)\\
&+r^2\big(1+24B\alpha\lambda+8A\alpha(1+\lambda)\big)\\
&+Br^6\big(A\lambda+B(2+4\lambda)\big)-r^4\big(A\big(2+(4+8B\alpha)\lambda\big)\\
&+B\big(3\lambda+4B\alpha(2+3\lambda)\big)\big)\big)\big)\big)<0, \qquad (3.1)
\end{aligned}
$$

and also Eq. (2.6) reveals that $\frac{dp_r}{dr}<0$, depicting the decrease in energy density and radial pressure with increasing radius of the compact object, as is well supported by the results shown in Figs. 4 and 5. Maximality of the central density and pressure is achievable at $r=0$, indicating

$$
\frac{d\rho}{dr}=0, \qquad \frac{dp_r}{dr}=0,
$$

$$
\frac{d^2\rho}{dr^2}<0, \qquad \frac{d^2p_r}{dr^2}<0.
$$

Using the EoS $p_r=\omega_r\rho$ and $p_t=\omega_t\rho$, we obtain the following form of the EoS parameters:

$$
\begin{aligned}
\omega_r=&\big(-e^{2Ar^2}\big(r^2+2\alpha(-1+\lambda)\big)+e^{Ar^2}\big(4\alpha(-1+\lambda)\\
&+ABr^6\lambda+r^2\big(1+8B\alpha(-1+\lambda)-8A\alpha\lambda\big)\\
&+Br^4(2+\lambda-8A\alpha\lambda+4B\alpha\lambda)\\
&-2\alpha\big(1+3Br^2+B^2r^4-Ar^2\big(2+Br^2\big)\big)
\end{aligned}
$$

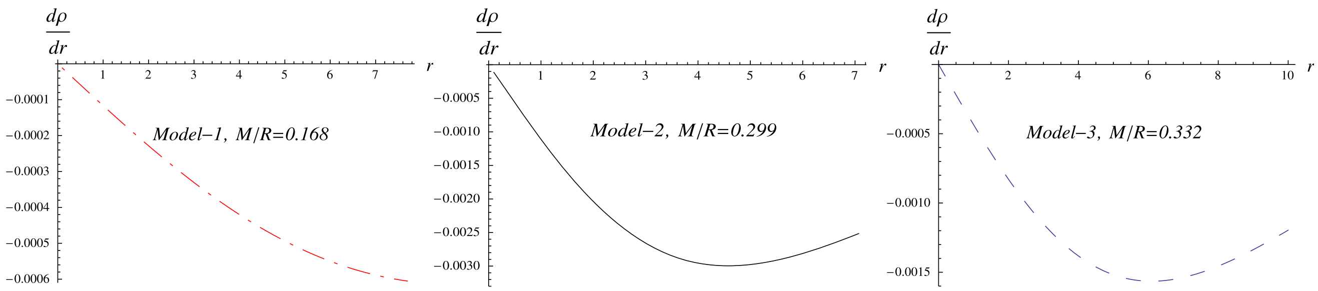

Fig. 4 Behavior of $\frac{d\rho}{dr}$ with increasing radius r

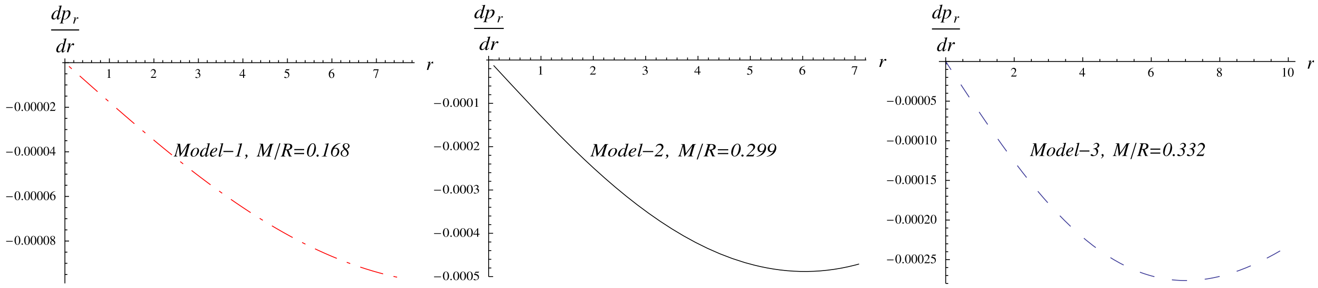

Fig. 5 Evolution of $\frac{dp_r}{dr}$ with increasing radius r

$$\begin{aligned}
&\times\left(-1+Br^2(-1+\lambda)+\lambda+B^2r^4(1+\lambda)\right.\\
&\left.-Ar^2\left(2(1+\lambda)+Br^2(1+3\lambda)\right)\right)))\\
&/\left(e^{2Ar^2}\left(r^2+2\alpha(-1+\lambda)\right)+2\left(-1-3Br^2-B^2r^4\right.\right.\\
&\left.+Ar^2\left(2+Br^2\right)\right)\alpha\left(-3Br^2(1+3\lambda)+1-\lambda\right.\\
&\left.+Ar^2\left(-2+Br^2\right)(1+3\lambda)+B^2r^4(3+7\lambda)\right)\\
&-e^{(Ar^2)}\left(4\alpha(-1+\lambda)+r^2\left(1+24B\alpha\lambda\right.\right.\\
&\left.+8A\alpha(1+\lambda)\right)+Br^6\left(A\lambda+B(2+4\lambda)\right)\\
&-r^4\left(A\left((4+8B\alpha)\lambda+2\right)\right.\\
&\left.+B\left(3\lambda+4B\alpha(2+3\lambda)\right)\right)))), \qquad (3.2)
\end{aligned}$$

$$\begin{aligned}
\omega_t=&\left(-2\alpha\left(-1-3Br^2-B^2r^4+Ar^2\left(2+Br^2\right)\right)\right.\\
&\times\left(-3-B^2r^4-2\lambda+Br^2\left(-1+Ar^2\right)(1+2\lambda)\right)\\
&+e^{2Ar^2}\left(r^2(2+\lambda)-2\alpha(3+2\lambda)\right)\\
&+e^{Ar^2}\left(4\alpha(3+2\lambda)+Br^6(A-B+A\lambda)\right.\\
&-r^2\left(2+\lambda+4A\alpha(3+2\lambda)-4B\alpha(5+4\lambda)\right)\\
&\left.\left.+r^4\left(B(-1+4B\alpha)(2+\lambda)+A\left(1-8B\alpha(1+\lambda)\right)\right)\right)\right)\\
&/\left(e^{2Ar^2}\left(r^2+2\alpha(-1+\lambda)\right)+2\alpha\left(-1-3Br^2-B^2r^4\right.\right.\\
&\left.+Ar^2\left(2+Br^2\right)\right)\left(1-\lambda-3Br^2(1+3\lambda)\right.\\
&\left.+Ar^2\left(-2+Br^2\right)(1+3\lambda)+B^2r^4(3+7\lambda)\right)\\
&-e^{(Ar^2)}\left(4\alpha(-1+\lambda)+r^2\left(1+24B\alpha\lambda\right.\right.\\
&\left.+8A\alpha(1+\lambda)\right)+Br^6\left(A\lambda+B(2+4\lambda)\right)\\
&-r^4\left(A\left(2+(4+8B\alpha)\lambda\right)\right.\\
&\left.\left.\left.+B\left(3\lambda+4B\alpha(2+3\lambda)\right)\right)\right)\right). \qquad (3.3)
\end{aligned}$$

It is interesting to mention here that the EoS parameters depend on the radius, rather than being a constant quantity as in an ordinary matter distribution. This non-constant behavior of the EoS parameters is constituted by the usual matter and exotic matter contributions (see Figs. 6 and 7).

The measure of anisotropy, $\Delta=\frac{(p_t-p_r)}{p_r}$, for the considered $f(R,T)$ model takes the following form:

$$\begin{aligned}
\Delta=&\frac{2e^{-2Ar^2}}{r^5(1+3\lambda+2\lambda^2)}\left[\left\{2\alpha\left(1+3Br^2+B^2r^4\right.\right.\right.\\
&\left.-Ar^2\left(2+Br^2\right)\right)\left(-3-B^2r^4-2\lambda\right.\\
&\left.+Br^2\left(Ar^2-1\right)(1+2\lambda)\right)+e^{2Ar^2}\left(r^2(2+\lambda)\right.\\
&\left.-2\alpha(3+2\lambda)\right)+e^{Ar^2}\left(4\alpha(3+2\lambda)\right.\\
&+Br^6(A-B+A\lambda)-r^2\left(2+\lambda+4A\alpha(3+2\lambda)\right.\\
&\left.-4B\alpha(5+4\lambda)\right)+r^4\left(B(4B\alpha-1)(2+\lambda)\right.\\
&\left.\left.\left.+A\left(1-8B\alpha(1+\lambda)\right)\right)\right)\right\}-\left\{e^{2Ar^2}\left(r^2+2\alpha(\lambda-1)\right.\right.
\end{aligned}$$

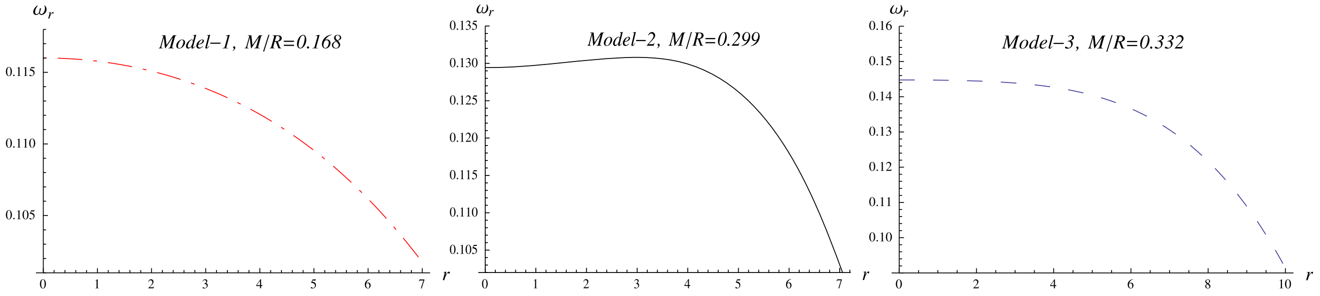

Fig. 6 The evolution of the radial EoS parameter across stars

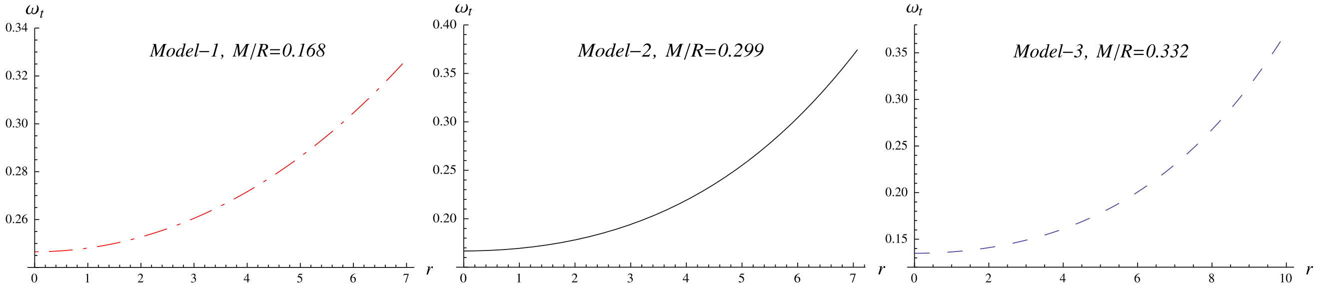

Fig. 7 The evolution of the tangential EoS parameter across compact stars

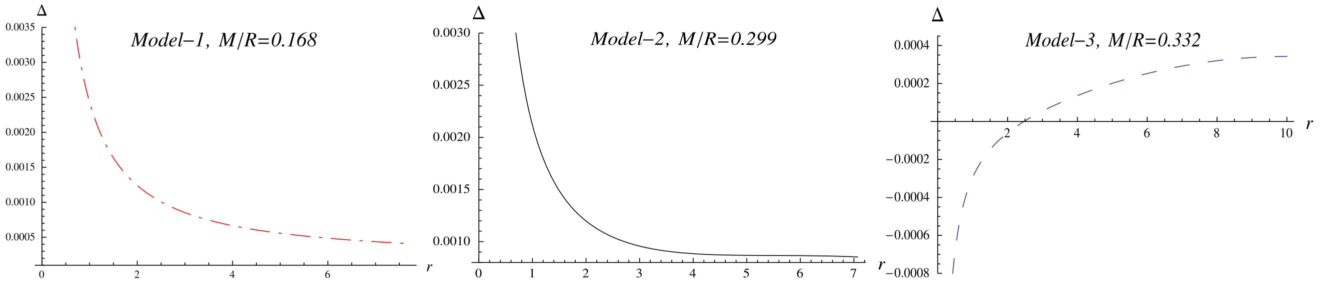

Fig. 8 Anisotropy measure Δ for compact stars

$$
\begin{aligned}
&+e^{Ar^2}\big(4\alpha(\lambda-1)+ABr^6\lambda+r^2\big(1+8B\alpha(\lambda-1)\\
&-8A\alpha\lambda\big)+Br^4\big(2+\lambda-4\alpha\lambda(2A-B)\big)\big)\\
&-2\alpha\big(1+3Br^2-Ar^2\big(2+Br^2\big)+B^3r^4\big)\\
&\times\big(-1+Br^2(\lambda-1)-Ar^2\big(2(1+\lambda)\\
&+Br^2(1+3\lambda)\big)\big)\big)\big\}\big]. \qquad (3.4)
\end{aligned}
$$

Figure 8 describes the evolution of the anisotropy measure, $p_t > p_r$, i.e., $\Delta > 0$ corresponds to the outward drawn anisotropy and its directed inward when $\Delta < 0$. In our model, we find that $\Delta > 0$ for the different compact stars, as shown in Fig. 8. It can be seen from Fig. 8 that $\Delta > 0$ at most of the points for compact stars of Model 1 and Model 3, indicating that a repulsive anisotropic force occurs, allowing the construction of more massive distributions. In our model, the anisotropy measure for Model 2 decreases with the increase in radius and becomes negative beyond $r = 2.8$ km.

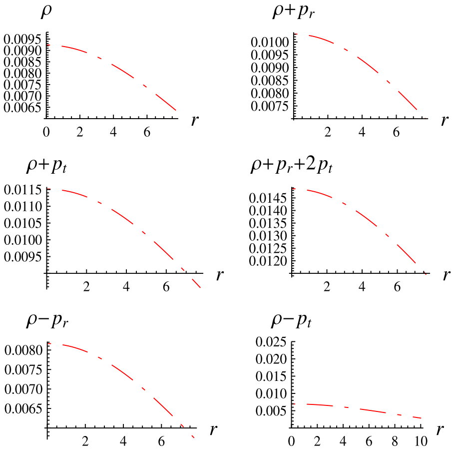

Fig. 9 Energy conditions for Model 1

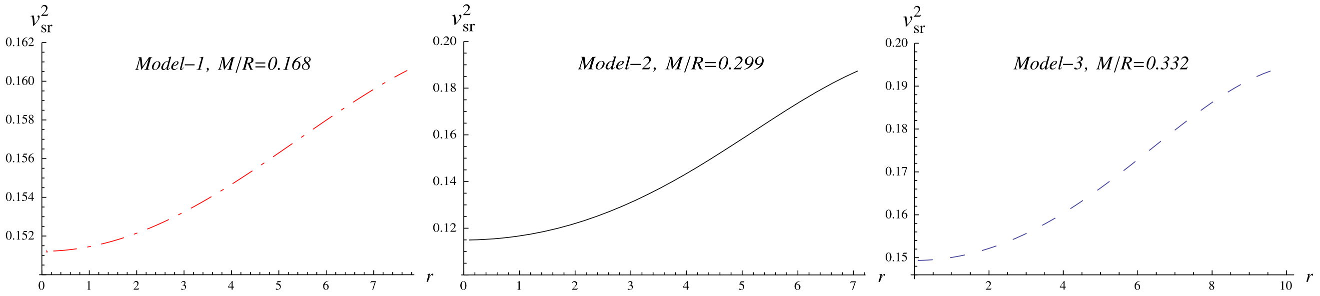

Fig. 10 Plot of v_{sr}^2 varying with radius r

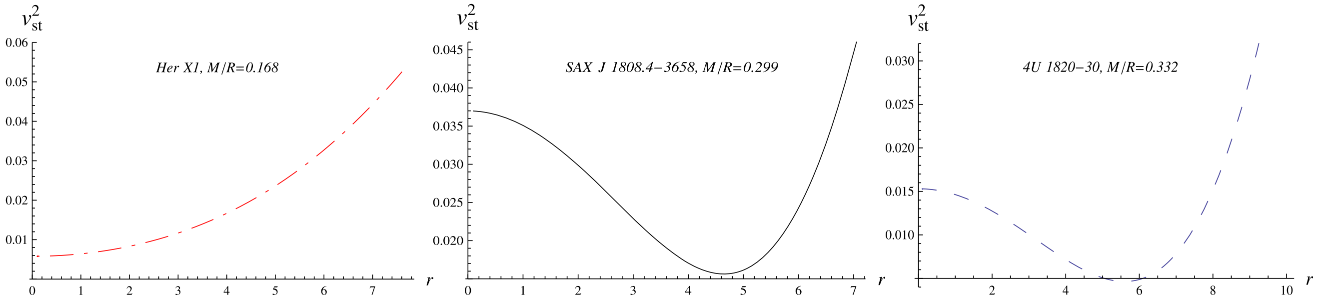

Fig. 11 Plot of v_{st}^2 varying with radius r

3.2 Energy conditions

Energy bounds are of significance; corresponding to the null energy condition (NEC), the weak energy condition (WEC), the strong energy condition (SEC), and the dominant energy condition (DEC), defined as

$$\begin{aligned}
\text{NEC}: &\quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \\
\text{WEC}: &\quad \rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \\
\text{SEC}: &\quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad \rho + p_r + 2p_t \geq 0, \\
\text{DEC}: &\quad \rho > |p_r|, \quad \rho > |p_t|.
\end{aligned}$$

The considered anisotropic sphere satisfies the energy conditions, exhibited graphically in Fig. 9 for a compact star of Model 1.

3.3 Causality conditions and stability analysis

The radial and transverse sound speeds, denoted by v_{sr} and v_{st}, should be less than the speed of light, i.e., $0 \leq v_{sr}^2 \leq 1$, $0 \leq v_{st}^2 \leq 1$, where $v_{sr}^2 = \frac{dp_r}{d\rho}$ and $v_{st} = \frac{dp_t}{d\rho}$. We plot the evolution of the radial and transverse sound speeds for compact stars and find that the above-mentioned conditions hold, as shown in Figs. 10 and 11.

One might proceed with the stability analysis of compact objects considering sound speeds (Herrera 1992; Herrera and Barreto 2013; Herrera and Santos 1997; Herrera et al. 2008). The potentially stable and unstable regimes can be estimated by considering the difference of the sound propagation within the matter distribution. The variation of $v_{st}^2 - v_{sr}^2$ of different strange stars is shown in Fig. 12. Figure 12 shows that the difference of the two sound speeds, i.e., $v_{st}^2 - v_{sr}^2$, retains a similar sign within the specific configuration and it satisfies the inequality $0 < |v_{st}^2 - v_{sr}^2| < 1$. Thus, our proposed compact stars models are stable.

4 Conclusion

A modified theory of gravity can provide an explanation of the accelerated expansion of universe. Modified gravity theories (such as $f(R,T)$ gravity) have been at the center of attention for many researchers in the recent past, because this type of theories seems to provide a viable explanation for dark energy. The conformal relation of $f(R,T)$ to general theory of relativity with a self-interacting scalar field has been examined (Astashenok et al. 2014).

This paper deals with the interior solutions for anisotropic fluids, which have been used to model compact stars in the context of a modified gravity theory $f(R,T)$. The modeling has been completed by taking compact stars as anisotropic in $f(R,T)$ gravity. For the exact solution of the governing differential equations, we have used the Krori and Barua (1975) form of the metric function, i.e., $b = Ar^2$, $a = Br^2 + C$, A, B, and C are arbitrary constants, which have been calculated by using some matching conditions. The smooth matching of interior and exterior regions of a star lead us to express the unknown constants in terms of masses and radii of the compact stars. Using the observed values of the masses and

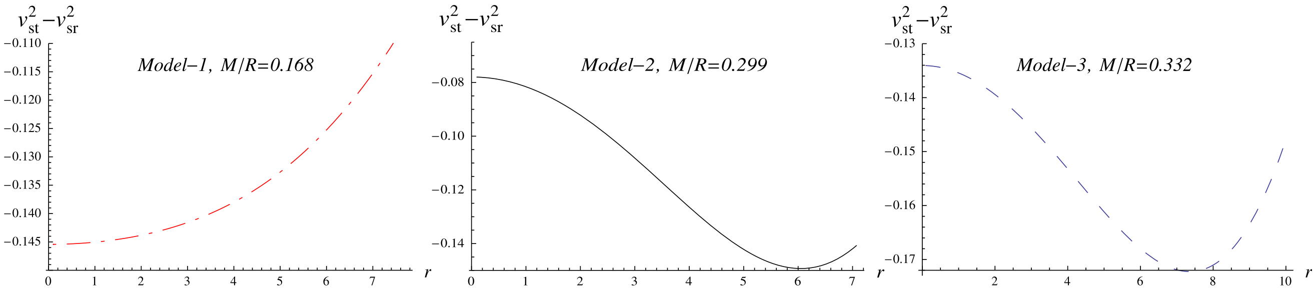

Fig. 12 Plot of $v_{st}^2 - v_{sr}^2$ for anisotropic compact stars

radii of the compact stars, we get the values of the model constants that are used to discuss the nature of the stars. For the calculated values of the constant, we found that the energy density and the radial and transverse pressure decrease for the given class of compact strange stars. For the particular choice $f(R,T)$, we have found that the EoS parameters behave like a normal matter distribution. On the basis of this fact, we may conclude that compact stars are composed of ordinary matter and the effect of the $f(R,T)$ term. The regularity analysis of the proposed model implies that the density and pressure are regular everywhere and attain the maximum value at the center. Thus the radial pressure p_r and the matter density ρ have maximum values at the center, decreasing from the center to the surface of the star, where the density becomes constant and the pressure reduces to zero.

It has been found that the anisotropy will be directed outward when $p_t > p_r$; this implies that $\Delta > 0$. We have found that $\Delta > 0$ for compact stars of Model 1 and Model 3, indicating that a repulsive anisotropic force occurs, allowing the construction of more massive distributions. However, the anisotropy measure for Model 2 decreases with the increase in radius and becomes negative beyond $r = 2.8$ km. The potentially stable and unstable regimes have been estimated by considering the difference of the sound propagation within the matter distribution. The compact stars remain potentially stable in the regions where the difference of radial and sound speeds remains positive. It is found that $0 < |v_{st}^2 - v_{sr}^2| < 1$ for all three considered compact stars. So, our considered model is potentially stable in $f(R,T)$ gravity, as shown in Fig. 12.

We would like to mention that TOV equations for modified theories of gravity like $f(G)$ (Momeni and Myrzakulov 2015), non-local $f(R)$ gravity (Momeni and Myrzakulov 2015) and $f(R,T)$ (Moraes et al. 2015) have been studied analytically and numerically.

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper.

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References

Abbas, G., Nazeer, S., Meraj, M.A.: Astrophys. Space Sci. **354**, 449 (2014)
Abbas, G., Kanwal, A., Zubair, M.: Astrophys. Space Sci. **357**, 109 (2015a)
Abbas, G., et al.: Astrophys. Space Sci. **357**, 158 (2015b)
Abbas, G., Qaisar, S., Meraj, M.A.: Astrophys. Space Sci. **357**, 156 (2015c)
Alavirad, H., Weller, J.M.: Phys. Rev. D **88**, 124034 (2013)
Arapoglu, S., Deliduman, C., Eksi, K.Y.: J. Cosmol. Astropart. Phys. **07**, 020 (2011)
Astashenok, A.V., Capozziello, S., Odintsov, S.D.: Phys. Rev. D **89**, 103509 (2014)
Astashenok, A.V., Capozziello, S., Odintsov, S.D.: J. Cosmol. Astropart. Phys. **01**, 001 (2015)
Azizi, T.: Int. J. Theor. Phys. **52**, 3486 (2013)
Baade, W., Zwicky, F.: Phys. Rev. **46**, 76 (1934)
Barrientos, O.J., Rubilar, F.G.: Phys. Rev. D **90**, 028501 (2014)
Cooney, A., et al.: Phys. Rev. D **83**, 064033 (2010)
Ganguly, A., et al.: Phys. Rev. D **89**, 064019 (2014)
Ghosh, P.: Rotation and Accretion Powered Pulsars. World Scientific, Singapore (2007)
Goswami, R., Nzioki, A.M., Maharaj, S.D., Ghosh, S.G.: Phys. Rev. D **90**, 084011 (2014)
Harko, T., Lobo, F.S.N.: Eur. Phys. J. C **70**, 373 (2010)
Harko, T., Lobo, F.S.N., Nojiri, S., Odinstov, S.D.: Phys. Rev. D **84**, 024020 (2011)
Herrera, L.: Phys. Lett. A **165**, 206 (1992)
Herrera, L., Barreto, W.: Phys. Rev. D **88**, 084022 (2013)
Herrera, L., Santos, N.O.: Phys. Rep. **286**, 53 (1997)
Herrera, L., Ospino, J., Di Prisco, A.: Phys. Rev. D **77**, 027502 (2008)
Hossein, S.K.M., et al.: Int. J. Mod. Phys. D **21**, 1250088 (2012)
Jamil, M., Momeni, D., Myrzakulov, R.: Chin. Phys. Lett. **29**, 109801 (2012a)
Jamil, M., Momeni, D., Raza, M., Myrzakulov, R.: Eur. Phys. J. C **72**, 1999 (2012b)
Kalam, M., et al.: Eur. Phys. J. C **72**, 2248 (2012)
Kausar, H.R., Noureen, I.: Eur. Phys. J. C **74**, 2760 (2014)
Krori, K.D., Barua, J.: J. Phys. A, Math. Gen. **8**, 508 (1975)
Lattimer, J.M., Steiner, A.W.: Astrophys. J. **784**, 123 (2014)
Li, X.D., Bombaci, I., Dey, M., Dey, J., van den Heuvel, E.P.J.: Phys. Rev. Lett. **83**, 3776 (1999)
Lobo, F.S.N.: Class. Quantum Gravity **23**, 1525 (2006)
Longair, M.S.: High Energy Astrophysics. Cambridge University Press, Cambridge (1994)
Maharaj, S.D., Sunzu, J.M., Ray, S.: Eur. Phys. J. Plus **129**, 3 (2014)
Mak, M.K., Harko, T.: Int. J. Mod. Phys. D **13**, 149 (2004)
Maurya, S.K., Gupta, Y.K.: Phys. Scr. **86**, 025009 (2012)
Maurya, S.K., Gupta, Y.K.: Astrophys. Space Sci. **344**, 243 (2013)

Maurya, S.K., Gupta, Y.K.: Astrophys. Space Sci. **353**, 657 (2014)
Momeni, D., Myrzakulov, R.: Int. J. Geom. Methods Mod. Phys. **12**, 1550014 (2015)
Momeni, D., Myrzakulov, R., Gudekli, E.: Int. J. Geom. Methods Mod. Phys. **12**, 1550101 (2015a)
Momeni, D., Gholizade, H., Raza, M., Myrzakulov, R.: Int. J. Mod. Phys. A **30**, 1550093 (2015b)
Moraes, P.H.R.S., Jose Arbanil, D.V., Malheiro, M.: arXiv:1511.06282 [gr-qc] (2015)
Noureen, I., Zubair, M.: Astrophys. Space Sci. **355**, 2202 (2014)
Noureen, I., Zubair, M.: Eur. Phys. J. C **75**, 62 (2015)
Noureen, I., Bhatti, A.A., Zubair, M.: J. Cosmol. Astropart. Phys. **1502**, 033 (2015)
Pant, N., Pradhan, N., Murad, H.M.: Astrophys. Space Sci. (2014a, accepted)
Pant, N., Pradhan, N., Singh, K.N.: J. Gravity **2014**, 380320 (2014b). doi:10.1155/2014/380320
Rahaman, F., et al.: Gen. Relativ. Gravit. **44**, 107 (2012a)
Rahaman, F., et al.: Eur. Phys. J. C **72**, 2071 (2012b)
Ruderman, R.: Astron. Astrophys. Rev. **10**, 427 (1972)
Shabani, H., Farhoudi, M.: Phys. Rev. D **90**, 044031 (2014)
Sharif, M., Zubair, M.: J. Cosmol. Astropart. Phys. **03**, 028 (2012a)
Sharif, M., Zubair, M.: J. Phys. Soc. Jpn. **81**, 114005 (2012b)
Sharif, M., Zubair, M.: J. Phys. Soc. Jpn. **82**, 014002 (2013)
Yazadjiev, S.S., Doneva, D.D., Kokkotas, K.D., Staykov, K.V.: J. Cosmol. Astropart. Phys. **06**, 003 (2014)