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Dynamic Modeling and Grid Integration of a Solid Oxide Fuel Cell

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***Abstract*--This article presents an equivalent dynamic model of a Solid Oxide Fuel Cell (SOFC) integrated with utility grid. The proposed dynamic model accounts for the electrochemical and thermodynamic characteristics of SOFC. Effects of temperature variations and fluid flow changes are incorporated in the model. Voltage drops inside a fuel cell are also modeled. This proposed model is further integrated with the utility grid with help of power conditioning unit (PCU). PCU consists of a DC-DC Boost converter and a 3-phase voltage source inverter. The output of 3-phase inverter is fed to the grid with the use of AC link integration. Complete model is formulated in MATLAB-SIMULINK environment. Simulation results clearly depict the behavior of SOFC and also show that SOFC is a potential source for distributed generation purpose. The proposed model formulated here would be helpful for the optimal prediction of characteristics and dynamic operation of SOFC based power plants.**

***Keywords:* Dynamic Model, Grid Integration, MATLAB-SIMULINK, PCU, SOFC**

NOMENCLATURE

E	Reversible potential of single cell [volts]
E_0	Reference voltage at STP [volts]
F	Faraday's constant [96487 C/mole]
I	Current [A]
j	Current density [A/m^2]
k_E	Constant for calculating E_0 [V/K]
k_i	Valve molar constant for specie I [mol/s atm]
m	Constant for calculating$_{trans}$[3×10^{-5}V]
n	Constant for calculating η_{trans} [8×10^{-4} m^2/A]
N_{cell}	Number of cells in FC stack
p_i	Partial pressure of specie i [atm]
q_{net}	Total heat energy [J]
q_i	Flow rate of specie I [mol/s]
R	Universal gas constant [8.3143 Jmol^{-1}K^{-1}]
R_i	Resistance of type i [Ω]
T	FC stack temperature [K]
$T_{initial}$	Initial temperature for FC stack [K]
V_{cell}	FC terminal voltage [V]
η_i	Voltage drop of type i [V]
τ_i	Response time for specie i [s]

SUBSCRIPTS AND SUPERSCRIPTS

a	Anode
act	Activation
c	Cathode
conc	Concentration
ohm	Ohmic

I. INTRODUCTION

SOFC is a static energy device that operates at high temperatures (800°C – 1000°C) and converts chemical energy into electrical energy. SOFC is now considered to be a promising technology in the field of distributed generation due to its high energy conversion efficiency and absence of moving parts.

They are more advantageous as compared to the conventional power plants; also they are promising in stationary power generation field [1], [2].

Modeling and computer simulation of fuel cells (FC's) has been much popular from past fifteen years. SOFC is a reliable energy source for steady state operations, but it cannot effectively respond to sudden load transients. The static and transient model of an SOFC, accounting for the effects of its thermal characteristics, mass flow rates and electrochemical reactions, is presented in [3]. A complete mathematical model of an SOFC is presented in [4]. The research articles mentioned above do not include the detail of the losses in SOFC which must be incorporated in the dynamic model for analyzing the actual behavior of SOFC over the time. Butler–Volmer equation has been used for approximation of activation losses in SOFC model presented in [5].

A dynamic model accounting for the losses in SOFC has been presented in [6].

Interconnection of a fuel cell to grid using different inverter topologies is presented in [7], [8]. Research work presented here, consists of a dynamic model of SOFC with a PCU, further connected to the utility grid. Simulations of the proposed model are carried out in MATLAB-SIMULINK environment.

This paper is organized as; section II and III deal with the formulation of the dynamic model of SOFC, section IV shows simulation results, section V and VI deal with the modeling of PCU and grid integration, section VII consists of conclusion.

II. SOFC MODEL FORMULATION

This section discusses a mathematical approach for modeling of SOFC. Following assumptions are made for the sake of simplification [9]:

1. Mathematical treatment is considered to be one dimensional.
2. Gasses are assumed to be ideal.
3. Constraints for the single cell can be lumped together to characterize a fuel cell stack.

A. *Partial Pressure Calculations*

The effective partial pressures of the reactant gasses must be known for calculating the output voltage of the FC. Partial pressure of hydrogen can be formulated by following the procedure discussed in [10], which is given as

$$p_{H_2} = \left(\frac{1/k_{H_2}}{1+\tau_{H_2}s}\right)\left(q_{H_2} - 2K_r I\right) \tag{1}$$

Where, K_r is a constant defined for calculating partial pressures and‘s’ is the Laplace variable. Similar procedure, as discussed in [10], is followed to calculate partial pressures of oxygen and water (steam).

$$p_{O_2} = \left(\frac{1/k_{O_2}}{1+\tau_{O_2}s}\right)\left(q_{O_2} - K_r I\right) \tag{2}$$

$$p_{H_2O} = \left(\frac{1/k_{H_2O}}{1+\tau_{H_2O}s}\right)\left(2K_r I\right) \tag{3}$$

B. *SOFC Output Voltage*

The overall SOFC reaction can be written as:

$$H_2 + 1/2O_2 = H_2O_{(g)}$$

Therefore, the corresponding Nernst equation can be written as

$$E_{cell} = E_0 + \frac{RT}{2F}\ln\left(\frac{p_{H_2} \cdot p_{O_2}^{1/2}}{p_{H_2O}}\right) \tag{4}$$

Where E_0 is expressed as [11]

$$E_0 = E_0^* - k_E(T-298) \tag{5}$$

Where, E_0^* is the standard voltage at 1-atm pressure and temperature of 298-K.

Equation (4) shows the open circuit voltage of SOFC. Under loaded condition, SOFC output voltage is less than the open circuit voltage due to some certain voltage drops inside the FC. These voltage drops are discussed and calculated in the following sub sections.

1) *Activation Voltage Drop*

Electrical energy is produced in FC’s due to chemical reactions. They must overcome activation energy barriers to precede the reactions. This leads to the activation voltage drop in the SOFC. The activation voltage drop is modeled using Butler-Volmer equation [12]

$$\eta_{act} = \frac{RT}{F}\left[\sinh^{-1}\left(\frac{j}{2j_{0,a}}\right) + \sinh^{-1}\left(\frac{j}{2j_{0,c}}\right)\right] \tag{6}$$

Where j is the FC current density, $j_{0,a}$is the exchange current density at anode and $j_{0,c}$ is the exchange current density at cathode. It is clear from (6) that activation over potential is zero in open circuit condition.

2) *Ohmic Voltage Drop*

Ohmic voltage drop is mainly due to resistance to the flow of ions in the ceramic electrolyte and the resistance to electrons through the electrodes and interconnections. At some constant temperature and geometry, ohmic voltage drop is proportional to the current. Ohmic over potential is given by Ohm’s law.

$$\eta_{ohm} = IR_{ohm} \tag{7}$$

Where, R_{ohm} is the sum of the sum of the anode R_a, electrolyte R_e, cathode R_c and interconnection resistance R_i.

$$R_{ohm} = R_a + R_e + R_c + R_i \tag{8}$$

The main cause of ohmic over voltage is due to the transport resistance offered by the electrolyte to the conducting ions. The resistance of electrolyte is mainly dependent upon temperature, and its effect cannot be neglected. Finally the ohmic voltage drop is formulated as [13]

$$\eta_{ohm} = I(R_{e0}e^{10100\left(\frac{1}{T}-\frac{1}{973}\right)} + R_a + R_c + R_i) \tag{9}$$

According to assumption 1, electrons and ions flow in SOFC is considered to be 1-dimensional (along z -axis). Constant values in the equation (10) are considered due to assumption 1. Putting the values of constant terms in the above expression, the final equation becomes

$$\eta_{ohm} = I(0.0248e^{10100\left(\frac{1}{T}-\frac{1}{973}\right)} + 0.0093) \tag{10}$$

3) *Concentration Voltage Drop*

This voltage drop is caused due to the change in concentration of reactants at the electrodes surface. At higher current densities, slow transportation of reactants to the reaction site is the main reason for the concentration voltage drop. This voltage drop is modeled [14] as

$$\eta_{conc} = m\exp(nI) \tag{11}$$

Where, m and n are constants for calculating concentration over potential. Values for these constants are 3x10^{-5}Vand 8x10^{-3}mA^{-1} respectively.

A. Practical Output Voltage

The practical output voltage for SOFC can be found by incorporating voltage drops calculated in the previous subsection.

$$V_{cell} = E_{cell} - \eta_{act} - \eta_{ohm} - \eta_{conc} \tag{12}$$

According to the assumption (3), the output FC stack voltage is calculated as:

$$V_{stack} = N_{cell} V_{cell} \tag{13}$$

B. SOFC Temperature Calculations

Heat generated by the FC can be used to calculate the change in temperature of SOFC. Further this change in temperature is further used for calculating the working temperature of SOFC. Total heat generated is formulated as [14]

$$q_{net} = nI(1.485 - V_{cell}) \tag{14}$$

$$\Delta T = \frac{q_{net}}{MC} \tag{15}$$

Where, M and $C$are the mass and specific heat energy of FC stack, whose values are taken to be 44kg and 560J/kg-K respectively. This change in temperature is used to calculate the working temperature of the FC stack.

$$T = T_{initial} + \Delta T \tag{16}$$

III. MODEL FORMULATION OF SOFC IN MATLAB/SIMULINK

The proposed model is built in MATLAB-SIMULINK environment, based upon the thermodynamic, chemical and electrical characteristics of the SOFC discussed and formulated in the above sections. Block diagram of the proposed dynamic model of SOFC is shown in Fig. 1. Various parameters used in the equations are mentioned in Appendix I. The only input to the model is the initial temperature. The working temperature of the FC is calculated at a given load current and voltage. The partial pressures of the hydrogen, oxygen and steam are found with the help of output current of the FC. Calculated load current, voltage and temperature are fed back to the respective blocks for calculating the different voltage drops in the FC discussed in the above sections. These calculated parameters are finally utilized to get thepractical output voltage.

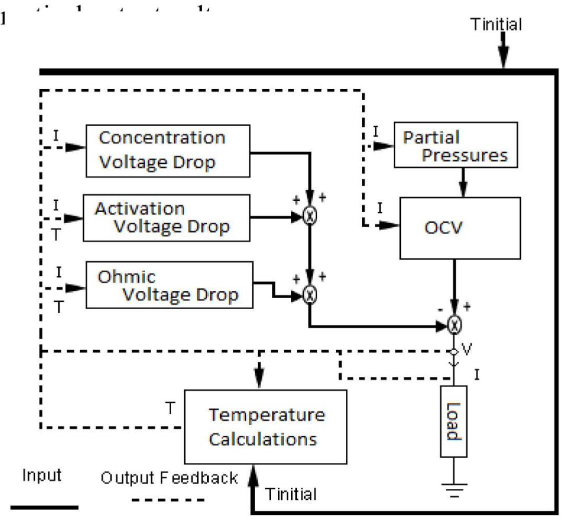

Figure 1. Block diagram of the proposed dynamic model

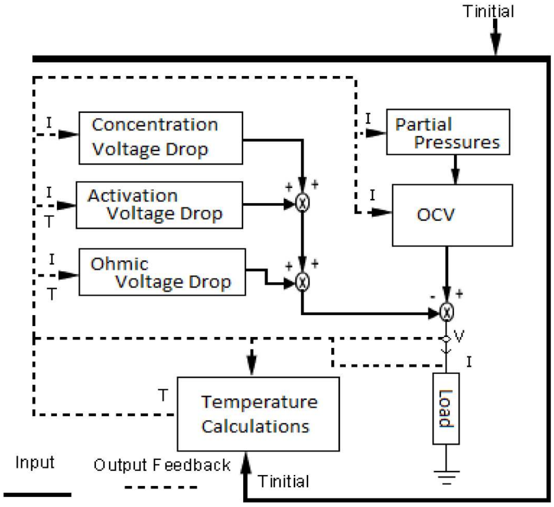

Figure 2. Block diagram of the proposed dynamic model

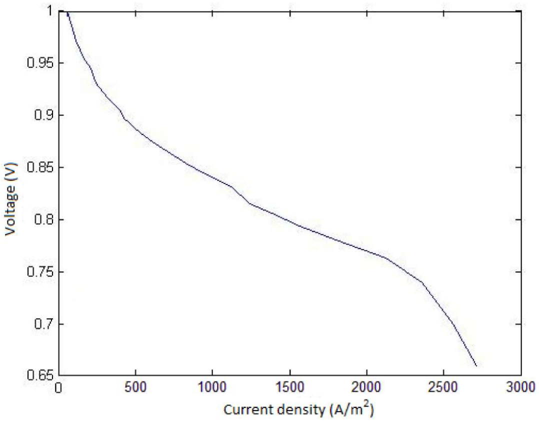

Figure 3. Polarization curve of the proposed model

IV. SIMULATION RESULTS OF THE PROPOSED SOFC MODEL

Simulation results of the proposed dynamic model are shown in this section. Simulation results show the exactbehavior of SOFC over the time. Simulation results consist of polarization curve (Voltage-current characteristics), power-current (P-I) characteristics and temperature response of the FC.

A. Polarization Curve

Fig. 2 shows the V-I characteristics (polarization curve) of the proposed dynamic model of SOFC. The

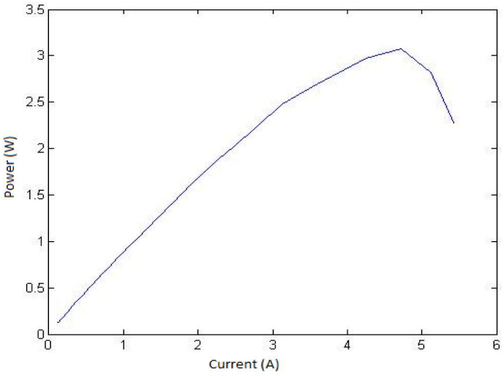

Figure 4. P-I Curve of the proposed model

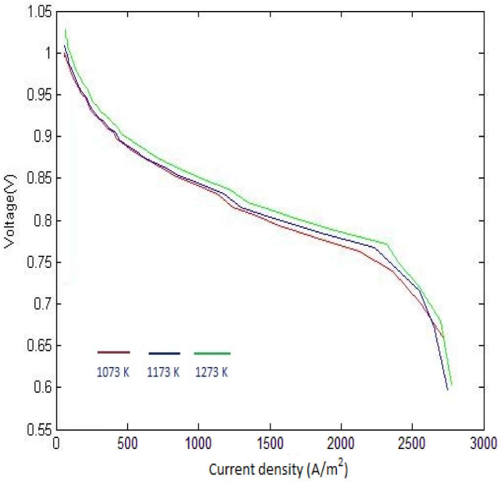

Figure 5. Temperature response of the model

result clearly exhibits the V-I response of SOFC. Voltage drop at the start of the curve is due to the activation over potential. At the mid region of the curve, voltage drop is due to the ohmic resistance. At higher current demand, the voltage falls suddenly due to mass transport losses (or concentration losses).

B. P-I Curve

Fig. 3 shows the P-I curve of SOFC, obtained from simulation results of the model. The result is in good correspondence with the published and experimental results. Fig. 3 depicts that the peak output power occurs at the rated output of SOFC. Beyond this limit SOFC goes into the concentration region, due to which FC output power decreases with increase in output current.

C. Temperature Response

Fig. 4 depicts the effect of change in operating temperature of SOFC on its output. Temperature is increased from 1073 K to 1273 K and its effect is monitored. From Fig. 4 it is clear that as the temperature is increased, the output voltage of SOFC is also increased. Activation and ohmic voltage drops are dependent on temperature and are the reason for this kind of response of SOFC. As temperature is increased the ohmic and activation voltage drops are reduced so output voltage is increased. The simulation result for the temperature response is in good correspondence with published and experimental results.

V. POWER CONDITIONING UNIT (PCU)

SOFC is a DC energy source that produces low voltage. Cells are connected in series to achieve high voltage. Further the stack voltage of SOFC is not suitable for grid connection. So a PCU is required for connecting SOFC stack with the utility grid. PCU consists of two modules: (i) a DC-DC boost regulator for stepping up the voltage of SOFC, (ii) a three phase DC-AC. the two modules of PCU will be discussed in the following subsections. Simulations are carried in MATLAB-SIMULINK environment.

A. Boost Converter

The DC output from SOFC is unregulated. The controlled DC-DC boost converter converts the unregulated DC into regulated DC output at the desired voltage level [15]. Along with regulated and controlled output, boost converter shows a fast response to the load variations than SOFC. A boost converter is designed and connected with the SOFC model. The output voltage of the proposed 25 kW SOFC stack model consisting of 424 cells, is boosted to 450 Volts using Pulse Width Modulation (PWM). Design of the buck-boost converter involves determination of the inductance (L), capacitance

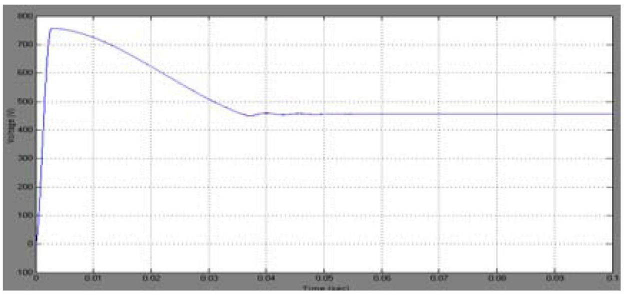

Figure 6. Output waveform of DC-DC Boost converter

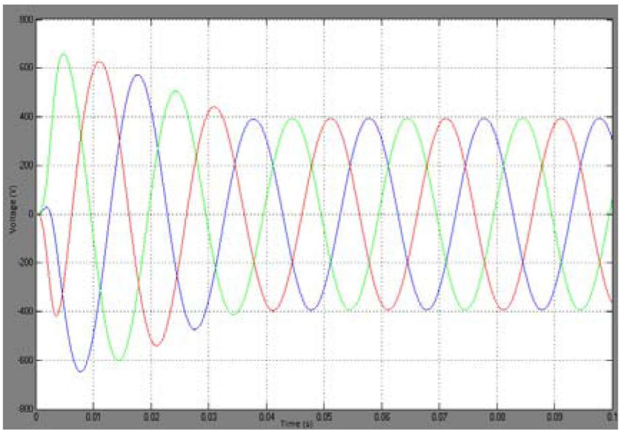

Figure 7. Output waveform of DC-AC Inverter

(C) and duty cycle (D). A filter inductor (L_f) is used in series with the load resistance for providing energy during the slow operation of SOFC and it also minimizes the ripples in output current. The values of L, C, D and L_f are given in the appendix II.

Fig. 5 shows the output of the boost converter. Voltage rises suddenly at the start of the simulation but after 0.035 seconds, the output of the boost converter becomes steady at the value of 450 V.

B. *Three Phase DC-AC Inverter*

SOFC stack with the DC-DC boost converter is further connected to the three phase DC-AC inverter. The output of the voltage source inverter (VSI) is connected to the grid. PWM is used to obtain the switching signals for MOSFETs. The output frequency of the 3 phase VSI is 50 Hz and output voltage is 390 V (line to line). Fig. 6 shows the output of VSI. At the start of simulation output voltage is quite high, which is due to the fact that output of boost converter is high at start and reaches at a steady value just after 0.035 sec, so the output of VSI also stabilizes at 0.035 sec after the start of simulation.

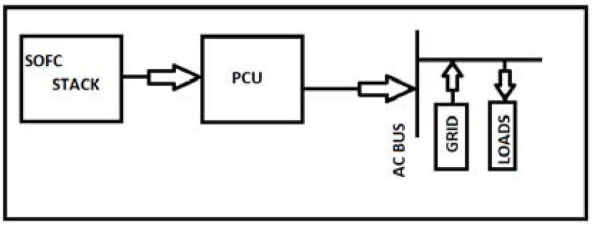

Figure 8. AC link grid integration of the proposed SOFC model

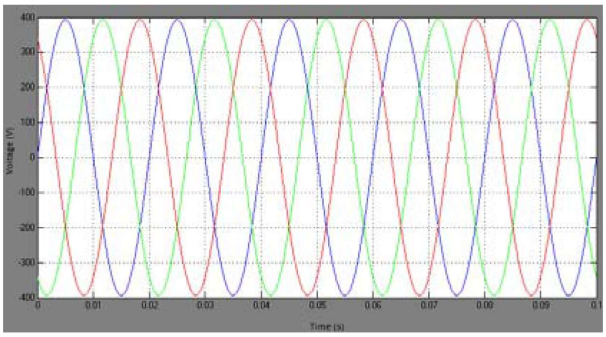

Figure 9. Voltage waveforms at grid side

VI. GRID INTEGRATION OF SOFC MODEL

This section deals with the integration of the proposed SOFC stack model with utility grid. SOFC stack is connected to a PCU, consisting of boost converter and a VSI, and then it is connected to the grid through AC link integration. The output from the VSI is integrated with the grid operating at 50 Hz and the voltage level at the point of integration is 390 V line to line. Block diagram of the proposed grid connected SOFC stack model is shown in Fig. 7. Main issues while using AC link integration are maintaining synchronism in frequency, voltage levels, and phase sequence of both sources at the time of interconnection and during operation. All of the three conditions are justified in the simulation as the voltage level; frequency and phase sequence are same of both VSI and grid. Voltage waveform at the AC bus of grid side is shown in the Fig. 8. Output of the VSI is distorted at start, but after 0.035 sec of simulation it becomes steady at 390 V (line to line). So, output of the VSI is connected to the grid after 0.035 sec to avoid any miss-matching. All simulations are carried out using MATLAB-SIMULINK tools.

The proposed model can also be connected to the grid AC bus at high voltages by simply stepping up the voltage at the inverter side with the help of step up transformer.

VII. CONCLUSIONS

An SOFC dynamic model is developed using MATLAB-SIMULINK tools. The proposed model accounts for the thermodynamic and electrochemical characteristics. The model also incorporates activation, ohmic and concentration voltage drops inside SOFC. Simulation results of the dynamic model shows the exact behavior of an SOFC. Temperature response of the model is also verified through simulation results.

The proposed SOFC stack model is connected to a PCU in order to connect it to the utility grid. PCU consists of a DC-DC boost converter to increase the voltage level of SOFC stack voltage. Boost converter also overcomes the effect of the slow response of SOFC due to the load variations. The output of boost converter is further connected to a VSI, which converts the DC voltage into 3 phase AC voltage at 50 Hz frequency. This output is further connected to the utility grid using AC link integration.

Output of the 3 phase VSI is distorted at the start of simulation but as the simulation time reaches 0.035 sec, its output becomes steady at 390 V approx. This variation is due to distortion in the output of DC-DC boost converter, which also becomes stable after 0.035 sec. Thus to avoid any miss-matching, the proposed model is integrated with the utility grid after 0.035 sec of the simulation time. Simulation results show that the frequency, voltage level and phase sequence of both sources, i.e. utility grid and proposed SOFC system, are same.

The proposed model shows that SOFC is a valuable source for the distributed generation purposes. The proposed model can be further used for the control based studies of the SOFC based power plants and its behavior can be checked under various fault conditions.

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APPENDICES

Appendix I: Parameters used in SOFC dynamic model

Symbol	Representation	Value
K_r	Constant	0.000996 (mol/s-A)
k_{H_2}	Valve molar constant for H_2	0.843 (mol/s-atm)
k_{O_2}	Valve molar constant for O_2	0.281 (mol/s-atm)
k_{H_2O}	Valve molar constant for H_2O	0.252 (mol/s-atm)
τ_{H_2}	Response time for H_2	26.1 s
τ_{O_2}	Response time for O_2	2.91 s
τ_{H_2O}	Response time for H_2O	78.3 s
q_{H_2}	H_2 flow rate	0.0864 (mol/s)
q_{O_2}	O_2 flow rate	0.07545 (mol/s)
$j_{0,a}$	Exchange current density at anode	5300 (A/m^2)
$j_{0,c}$	Exchange current density at cathode	2000 (A/m^2)

Appendix II: Parameters used in DC-DC Boost Converter

Symbol	Representation	Value
L	Inductance	90 µH
C	Capacitance	5000 µF
L_f	Filter Inductance	0.1 H
f	Switching Frequency	10 kHz