

Topological indices of line graphs of some convex polytopes and line graphs of their subdivisions



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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

*In the name of Allah
the most Gracious
the most Compassionate*



DECLARATION

I ***Fatima Asif*** D/O ***Muhammad Asif*** ID: ***15007109002***, Session 2015-2017 hereby declare that the matter printed in the thesis titled “**Topological indices of line graphs of some convex polytopes and line graphs of their subdivisions**” is my own work and has not been printed, published and submitted as research work, thesis or publication in any form in any University, Research institution etc. in Pakistan or Abroad.

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Abstract

In this thesis, we give theoretical results for some topological indices such as geometric-arithmetic index $GA(G)$, fifth geometric-arithmetic index $GA_5(G)$, atom-bond connectivity index $ABC(G)$, fourth atom-bond connectivity index $ABC_4(G)$, Zagreb indices $M_1(G)$, $M_2(G)$, $M_3(G)$, $M_1(\overline{G})$, $M_2(\overline{G})$, Zagreb coindices $\overline{M}_1(G)$, $\overline{M}_2(\overline{G})$, $\overline{M}_2(G)$, Randic index $R(G)$, hyper-Zagreb index $HM(G)$, general sum-connectivity index $\chi(G)$, augmented Zagreb index $AZI(G)$ by considering graph G as a line graph and line graph of the subdivision of some convex polytopes.

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Bibliography

Introduction

Euler (1701-1783) is considered to be the father of graph theory with his work towards the solution of the problems in geometry (see [10, 13, 39]). In the field of chemistry, graph theory has been practiced to a vast range of research areas: quantum chemistry, synthetic chemistry and thermochemistry etc. Usually, graphs are categorized into structural graphs, molecular graphs and reaction graphs etc. Some of the graph theory concepts corresponds to the terms in chemistry e.g. point as an atom, line as a covalent bond, degree as atom valency and path as chemical substructure etc. Trinajstić and Gutman suggested that mathematical chemistry is that branch of theoretical chemistry that is associated with applications of mathematical techniques to solve the problems in chemistry. One can implement graph theory to numerical modeling of chemical process. Chemists utilize different classifications and methods