On metric dimension of flower graphs $f_{n\times m}$ and convex polytopes*

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Abstract. Let G be a connected graph and $d(x, y)$ be the distance between the vertices x and y. A subset of vertices $W = \{w_1, w_2, \cdots, w_k\}$ is called a resolving set for G if for every two distinct vertices $x, y \in V(G)$, there is a vertex $w_i \in W$ such that $d(x, w_i) \neq d(y, w_i)$. A resolving set containing a minimum number of vertices is called a metric basis for G and the number of vertices in a metric basis is its metric dimension $dim(G)$.
Let $\mathcal{F}$ be a family of connected graphs $G_n : \mathcal{F} = (G_n)_{n\geq 1}$ depending on n as follows: the order $|V(G)| = \varphi(n)$ and $\lim\limits_{n\to\infty} \varphi(n) = \infty$. If there exists a constant $C > 0$ such that $dim(G) \leq C$ for every $n \geq 1$ then we shall say that $\mathcal{F}$ has bounded metric dimension; otherwise $\mathcal{F}$ has unbounded metric dimension. If all graphs in $\mathcal{F}$ have the same metric dimension (which does not depend on n), then $\mathcal{F}$ is called a family with constant metric dimension.
The metric dimension of some classes of plane graphs has been determined in [3], [4], [5], [10], [12], [15] and [22], while metric dimension of some classes of convex polytopes has been studied in [10]. In this paper this study is extended, by considering flower graphs $f_{n\times m}$ and two classes of graphs associated to convex polytopes.

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