

Dynamical analysis of compact objects IN $F(R)$ and
 $F(R,T)$ theories

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I, **Ms. Ifra Noureen**, hereby declare that the matter printed in this thesis is my original work. This dissertation does not contain any material that has been submitted for the award of any other degree in any university and to the best of my knowledge, neither does this dissertation contain any material published or written previously by any other person, except due reference is made in the text of this dissertation.

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DEDICATED TO
My Father Muhammad Anwar Bajwa
and
My Mother Naseeb Kausar

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Abstract

Dynamical Analysis of Compact Objects in $f(R)$ and $f(R, T)$ Theories

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The astrophysics and astronomical theories are invigorated largely by the gravitational evolution and instability range explorations of gravitating sources. Gravitational collapse is the fundamental phenomenon to account evolution within galaxies and assemble supergiant structures. This dissertation is based on the explorations regarding dynamical instability of gravitating sources in $f(R)$ and $f(R, T)$ theories of gravity. The considered modified gravitational theories provide dark energy substitutes constituting large negative pressure and thought to be responsible for the cosmic speed-up.

The dynamical systems are studied by considering spherically and axially symmetric backgrounds with anisotropic matter distribution. The modified field equations and conservation equations for spherically symmetric dynamical system are constructed in $f(R, T)$ gravity. The variations in gravitating system are estimated by implementation of first order perturbations on dynamical equations. Insertion of perturbed physical quantities derived from perturbed field equations in perturbed Bianchi identities leads to the evolution equation. The expression for adiabatic index is constructed from evolution equation to investigate the variation in pressure stresses with the given energy density. Moreover, terms lying in Newtonian and post Newtonian eras are identified to establish the corrections to weak field limit.

We have also studied the dynamics of spherically symmetric anisotropic stars under the influence of shear-free condition. The modified field equations accompanying vanishing shear scalar are obtained. On establishment of evolution equation, it is observed that the flow variables are less constrained in shear-free case and so leads to a wider range of stability. The

corrections to Newtonian and post Newtonian approximations are estimated as well.

The dynamics of spherical stars evolving under expansion-free condition in $f(R, T)$ gravity is explored by taking anisotropic matter configuration. The collapse equation is acquired from linearly perturbed dynamical equations. It is concluded that in zero-expansion case, the unequal stresses and density profile defines instability range rather than the adiabatic index. However, the physical quantities are constrained to maintain positivity of energy density and stable stellar configuration.

Motivating from the incidental deviations from spherical symmetry of gravitating objects, we study the dynamical instability of axially symmetric sources (avoiding reflection and rotation terms about symmetry axis). Furthermore, the evolution equation is settled for both the considered modified theories leading to the instability range of axially symmetric dynamical system in Newtonian and post Newtonian regimes.

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Chapter 1

Introduction

The evolution of stars influenced by strong gravitational pull is extensively anticipated field in gravitational theories, carrying investigation of diverse characteristics of gravitating sources. Internal fusion of hydrogen to helium and then to other heavy elements within stars produce internal energy allowing them to radiate energy in the form of light. The stars remain stable as long as the balance is maintained between inward acting gravity force and outward drawn pressure produced by internal fusion processes.