

# Frequency Synchronization Algorithms for 802.11n based MIMO-OFDM Systems: A Performance Analysis

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**Abstract**—Orthogonal Frequency Division Multiplexing (OFDM) is a bandwidth efficient signaling scheme for digital communications that was first proposed by Chang. In OFDM, the spectrum of individual orthogonal subcarriers mutually overlaps, giving therefore an optimum spectrum efficiency. A data communications system based on OFDM is particularly sensitive to frequency offset in the channel. Frequency offset is a phenomena, which causes phase shift and hence, reduction in signal strength at the output of the filters matched to each of the carriers and introduces ICI from other carriers which are now no longer orthogonal to the filter. In OFDM the carriers are inherently closely spaced in frequency compared to the channel bandwidth, the tolerable frequency offset becomes a very small fraction of the channel bandwidth. This paper is based on the study and comparison of algorithms that estimate the carrier and sampling frequency offsets from the demodulated data signals in the receiver. An implementation of these techniques to correct the respective offsets along with the performance of these algorithms, via simulation, is also presented.

**Keywords** Orthogonal frequency division multiplexing, Multiple input-Multiple output systems, 802.11n, carrier frequency offset, frequency synchronization.

## I. INTRODUCTION

In a Multiple-Input-Multiple-Output (MIMO) system [1], [2] multiple antennas are used at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or transmit power. It achieves this by higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading). Because of these properties, MIMO is a current theme of international wireless research.

Orthogonal frequency-division multiplexing (OFDM) [1], [3] is a frequency-division multiplexing (FDM) scheme utilized as a digital multi-carrier modulation method. A large number of closely-spaced orthogonal sub-carriers are used to carry data. The data is divided into several parallel data streams or channels, one for each sub-carrier. Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation or phase-shift keying) at

a low symbol rate, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth. The primary advantage of OFDM over single-carrier schemes is its ability to cope with severe channel conditions (for example, attenuation of high frequencies in a long copper wire, narrowband interference and frequency-selective fading due to multipath) without complex equalization filters. Channel equalization is simplified because OFDM may be viewed as using many slowly-modulated narrowband signals rather than one rapidly-modulated wideband signal. The low symbol rate makes the use of a guard interval between symbols affordable, making it possible to handle time-spreading and eliminate Inter-Symbol Interference (ISI). This mechanism also facilitates the design of Single Frequency Networks (SFNs), where several adjacent transmitters send the same signal simultaneously at the same frequency, as the signals from multiple distant transmitters may be combined constructively, rather than interfering, as would typically occur in a traditional single-carrier system.

In OFDM, the sub-carrier frequencies are chosen so that the sub-carriers are orthogonal to each other [4], [5]. This greatly simplifies the design of both the transmitter and the receiver; unlike conventional FDM, a separate filter for each sub-channel is not required. The orthogonality also allows high spectral efficiency, with a total symbol rate near the Nyquist rate for the equivalent baseband signal (i.e., near half the Nyquist rate for the double-side band physical passband signal). Almost the whole available frequency band can be utilized. OFDM generally has a nearly 'white' spectrum, giving it benign electromagnetic interference properties with respect to other co-channel users.

## II. SYNCHRONIZATION OF MIMO-OFDM SYSTEMS

Synchronization of MIMO-OFDM systems can be categorized as follows:

- Timing Synchronization
- Carrier Frequency Offset Synchronization
- Sampling Frequency Offset Synchronization

This paper addresses only Carrier Frequency offset and its compensation algorithms. Various synchronization algorithms

have been discussed in [2], [3], [4], [5], [6], [7], [8]. We however discuss and compare the performances of the algorithms discussed in [9] and [10] for 802.11n systems. In an orthogonal frequency-division multiplexing system, synchronization at the receiver is one important step that must be performed. To acquire synchronization for either a continuous stream of data as in a broadcast application or for bursty data as in a WLAN. In both cases the receiver must continuously scan for incoming data, and rapid acquisition is needed. The ratio of the number of overhead bits for synchronization to the number of message bits must be kept to a minimum, and low-complexity algorithms are needed. Synchronization of an OFDM signal requires finding the symbol timing and carrier frequency offset. Symbol timing for an OFDM signal is significantly different than for a single carrier signal since there is not an “eye opening” where a best sampling time can be found. Rather there are hundreds or thousands of samples per OFDM symbol since the number of samples necessary is proportional to the number of subcarriers. Finding the symbol timing for OFDM means finding an estimate of where the symbol starts. There is usually some tolerance for symbol timing errors when a cyclic prefix is used to extend the symbol. Synchronization of the carrier frequency at the receiver must be performed very accurately, or there will be loss of orthogonality between the sub-carriers. OFDM systems are very sensitive to carrier frequency offsets since they can only tolerate offsets which are a fraction of the spacing between the subcarriers without a large degradation in system performance.

Rest of this paper is organized as following: section III explains the possible causes and effects of frequency offsets in OFDM-MIMO systems. Section IV explains the concept of Carrier Frequency offset (CFO) and presents two algorithms for CFO compensation. Section V gives the simulations and results of the two algorithms for CFO compensation and section VI concludes the paper.

### III. FREQUENCY OFFSETS: CAUSES & EFFECTS

Two types of frequency offsets are encountered in OFDM systems:

- Carrier Frequency Offset
- Sampling Frequency Offset

#### A. Carrier Frequency Offset (CFO)

The baseband signal at the receiver is multiplied by a carrier signal for the purpose of frequency up-conversion [4], [5], [6], [7], [8]. At the receiver, the corresponding frequency down-conversion is necessary. But since these signals are generated by local oscillators which are independent for both the transmitter and the receiver and are seldom synchronized. Therefore, this offset in the frequency of the receiver local oscillator causes degradation in the signal. Let  $x(t)$  be the transmitters baseband signal and let  $f_c$  be the carrier frequency, then the transmitted signal  $s(t)$  is given by:

$$s(t) = x(t) \cdot e^{j2\pi f_c t} \quad (1)$$

The received signal is then multiplied by the same sinusoidal and low-pass filtered to extract the original signal which is given by

$$r(t) = s(t) \cdot e^{-j2\xi f_c t} \quad (2)$$

Where  $\xi$  is the carrier frequency offset. Hence the effect of Carrier Frequency Offset (CFO) is equal to multiplication by a complex exponential in the baseband. The frequency offset correction is done in two parts, coarse and fine, and takes advantage of the fact that the short and long preambles are patterns that repeat. Both the coarse and fine estimates use the same method on the short and long preamble, respectively. The method is to correlate the early part of the preamble with the later [9]. The result is as follows:

$$\begin{aligned} R(\tau) &= \int r(t) \cdot r^*(t - \tau) \\ &= \|x(t)\|^2 \cdot e^{-2\pi\xi\tau} \end{aligned} \quad (3)$$

Taking the phase of  $R(\tau)$  will give the value of  $\xi$ . The same procedure is repeated for the two preambles to get the estimate of frequency offset.

The effect of Carrier Frequency Offset has been shown in the figs. 1, 2 and 3 In fig. 1, an array of elements has been transmitted, with 0s and 1s in it.

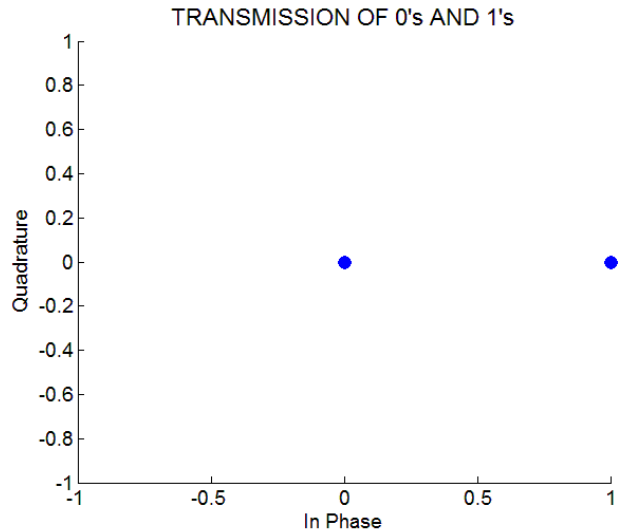


Fig. 1. Transmission of array of 0s and 1s

Now with different amount of CFO added we can clearly see the spreading and offset present, in figs. 2 and 3.

The scatter plot of received data, after passing through the channel, has been observed to undergo the following changes:

- A slight reduction in the signal amplitude.
- Rotation of the constellation due to the phase shift.

The DFT of transmitted symbol consists of complex sinusoids which have been up-converted with complex modulation values. The individual sinusoids are orthogonal on the symbol interval. After passing through a bandpass channel, the complex envelope of the received sequence consists of the effect of the transfer function of the channel at each frequency. At the

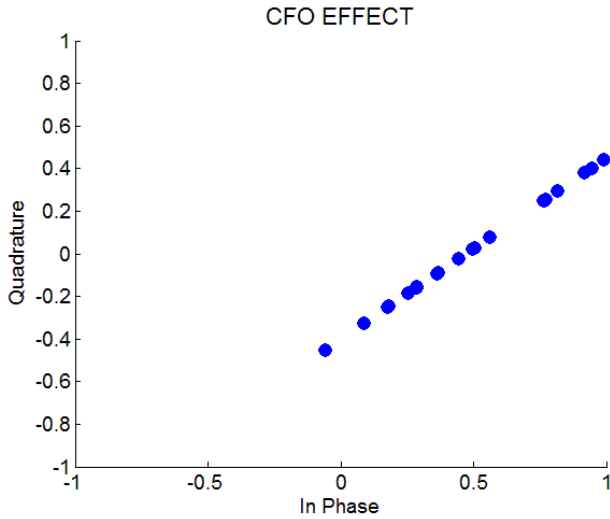


Fig. 2. Effects of CFO

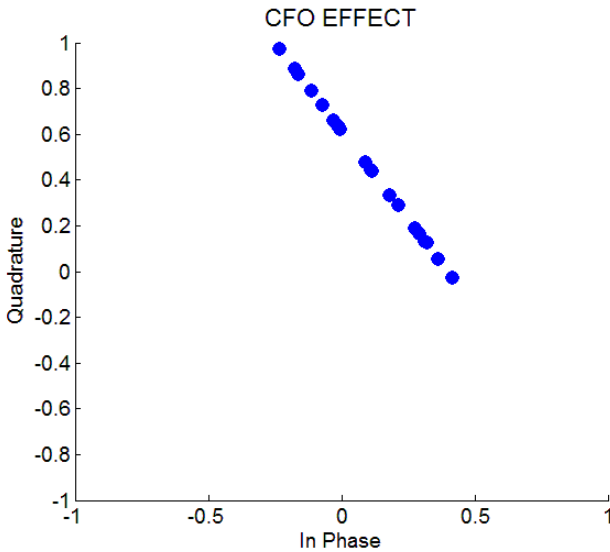


Fig. 3. Effects of CFO

receiver side, the demodulation process which is implemented by the DFT is affected by the frequency offset, and each element of the DFT sequence consists of the modulation value modified by the channel transfer function. This component experiences reduction in amplitude and phase shift due to the CFO which causes the rotation of the constellation diagram [9], [10].

### B. Sampling Frequency Offset (SFO)

SFO occurs when the clock speed of the receivers ADC is not matched to that of the transmitters DAC. This induces a phase shift on each carrier that is proportional to the carrier number and increases linearly with the symbol number. It causes rotation of the data symbols. Fig 4 explains the effects of Sampling Frequency.

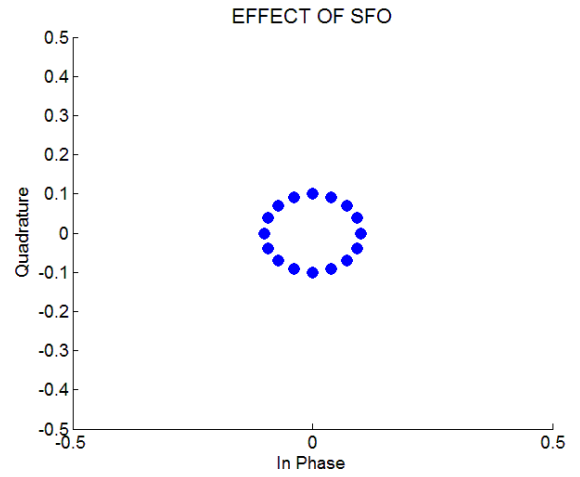


Fig. 4. Effect of SFO

## IV. SYNCHRONIZATION ALGORITHMS FOR CARRIER FREQUENCY OFFSET (CFO)

In this paper we have studied and implemented the following algorithms for the carrier frequency offset:

- A technique of orthogonal frequency division multiplexing frequency offset correction (Maximum Likelihood Estimation) [9].
- Robust frequency and timing synchronization for OFDM [10].

### A. Maximum Likelihood Estimation (MLE) [9]

The frequency offset estimation technique of [9] is based on the fact that if an OFDM symbol is repeated, one receives, in the absence of noise, the  $2N$  point sequence, the DFT of the first half of which differs from the second half by a phase of  $2\pi\xi$  where  $\xi$  is the frequency offset.

$$r_n = \frac{1}{N} \sum_{-K}^K X_K H_K e^{2\pi j n \frac{(k+\xi)}{N}} \quad (4)$$

where  $X_K$  and  $H_K$  are the transforms of input and channel respectively and  $r_n$  is the output. This algorithm assumes that the channel impulse response does not change (much) during the symbol plus guard interval and this corresponds to slow fading in radio frequency channel. It involves the repetition of data symbols and comparison of the phases of each of the carriers between the successive symbols [9]. Since the modulation phase values are not changed, the phase shift of each of the carriers between successive repeated symbols is due to the frequency offset. Between the first and second half of DFT the ICI and signal are altered in the same way, by a phase shift proportional to the frequency offset. Therefore, if offset  $\xi$  is estimated using these observations it is possible to obtain accurate estimates even when the offset is too large for satisfactory data demodulation. The frequency offset is estimated using a maximum likelihood estimate (MLE) algorithm [9]. The  $k$ th element of the  $N$  point DFT of the first  $N$  points of above equation is

$$R_{1k} = \sum_{n=0}^{N-1} r_n e^{-j2\pi \frac{kn}{N}} \quad (5)$$

and the  $k$ th element of the DFT of the second half of the sequence is

$$R_{2k} = \sum_{n=N}^{2N-1} r_n e^{-j2\pi \frac{kn}{N}} \quad (6)$$

since

$$r_{n+N} = r_n e^{2\pi j\xi} \quad (7)$$

hence

$$R_{2k} = R_{1k} e^{2\pi j\xi} \quad (8)$$

Including the AWGNs we obtain

$$Y_{1k} = R_{1k} + W_{1k} \quad (9)$$

and

$$Y_{2k} = R_{2k} e^{2\pi j\xi} + W_{2k} \quad (10)$$

Observe that between the first and second DFTs, both the ICI and the signal are altered in exactly the same way, by a phase shift proportional to frequency offset [9]. Therefore, if offset  $\xi$  is estimated using observations it is possible to obtain accurate estimates even when the offset is too large for satisfactory data demodulation. The maximum likelihood estimate of  $\xi$  is given by [9]

$$\xi = \frac{1}{2\pi} \arctan \frac{\sum_{k=-K}^K \text{Im}\{Y_{2k} \cdot Y_{1k}^*\}}{\sum_{k=-K}^K \text{Re}\{Y_{2k} \cdot Y_{1k}^*\}} \quad (11)$$

The flowchart in fig. 5 explains the algorithm.

Performance of the algorithm as a function of  $E_c/N_o$  (individual OFDM carrier energy to one-sided spectral density of additive white Gaussian noise) and BER will be simulated as a result of this paper. The limits of the accuracy of the estimation are  $\|\xi\| \leq 0.5$  that is half of the intercarrier spacing. When  $\|\xi\| \geq 0.5$  the estimate is no longer unbiased and in practice it becomes useless [9]. Thus for frequency offsets exceeding one half of the carrier spacing, an initial acquisition strategy must be prescribed that brings the offset within the limits of the algorithm. The basic strategy for initial carrier frequency offset acquisition, in either continuous or bursty data is to shorten the length of DFTs and use larger carrier spacing such that the phase shift does not exceed  $\pi$  [9]. The frequency offset in Hz is:

$$\begin{aligned} \delta &= \frac{\xi}{T} \\ &= \xi \Delta f \end{aligned} \quad (12)$$

where  $\Delta f$  is the intercarrier spacing and  $T$  is the symbol interval. Let us assume that the initial frequency offset is no greater than  $\delta_{max}$ . Then  $\Delta f_{initial} \geq 2\delta_{max}$  determines the minimum initial carrier spacing, and corresponding DFT

lengths. Correction is done by the multiplication of data symbols with  $e^{-j2\pi\xi t}$  [9].

### B. Robust Frequency and Timing Synchronization [10]

The authors of [10] introduced a method in which acquisition is achieved in two steps through the use of a two-symbol training sequence, which will usually be placed at the start of the frame. First the symbol/frame timing is found by searching for a symbol in which the first half is identical to the second half in time domain. Then the carrier frequency offset is partially corrected, and a correlation with a second symbol is performed to find the carrier frequency offset. The symbol timing recovery relies on searching for a training symbol with two identical halves in time domain, which will remain identical after passing through the channel, except that there will be a phase difference between them caused by the carrier frequency offset. Considering the first training symbol where the first half of the symbol is identical to the second half, the conjugate of a sample from the first half is multiplied by the corresponding sample from the second half and the result will have a phase of approximately  $\pi\Delta f$  [10]. The correlation window slides along the length of the symbols and the timing metric corresponding to this correlation has to be maximized. The index corresponding to the maximum of the metric is taken as the best timing point and the phase of the correlation metric at this timing point gives the estimate of frequency offset which is in fact the estimate of the fractional part of CFO [10].

Assume that timing synchronization has already been done, we obtain the following equation after timing synchronization:

$$P(d) = \sum_{m=0}^{L-1} r_{d+m}^* \cdot r_{d+m+L} \quad (13)$$

Thus the two halves of the first training symbol will be a phase difference of

$$\phi = \pi T \Delta f \quad (14)$$

Which can be estimated near the best timing point by

$$\hat{\phi} = \angle P(d) \quad (15)$$

If  $\|\phi\|$  can be guaranteed to be less than  $\pi$ , then the frequency offset estimate is

$$\hat{\Delta f} = \frac{\hat{\phi}}{\pi T} \quad (16)$$

Otherwise, the actual frequency offset estimate is given by:

$$\Delta f = \frac{\phi}{\pi T} + \frac{2z}{T} \quad (17)$$

where  $z$  is an integer. By partially correcting the frequency offset by multiplication of the data symbols with  $e^{-j2t\frac{\phi}{T}}$  ACI (Adjacent Carrier Interference) can be eliminated. Then the integer part estimation of CFO is done [10]. This is done by the use of a second training symbol and correlating the first half of the symbol with the second half over a set of CFO estimated integer values in frequency domain. The integer corresponding

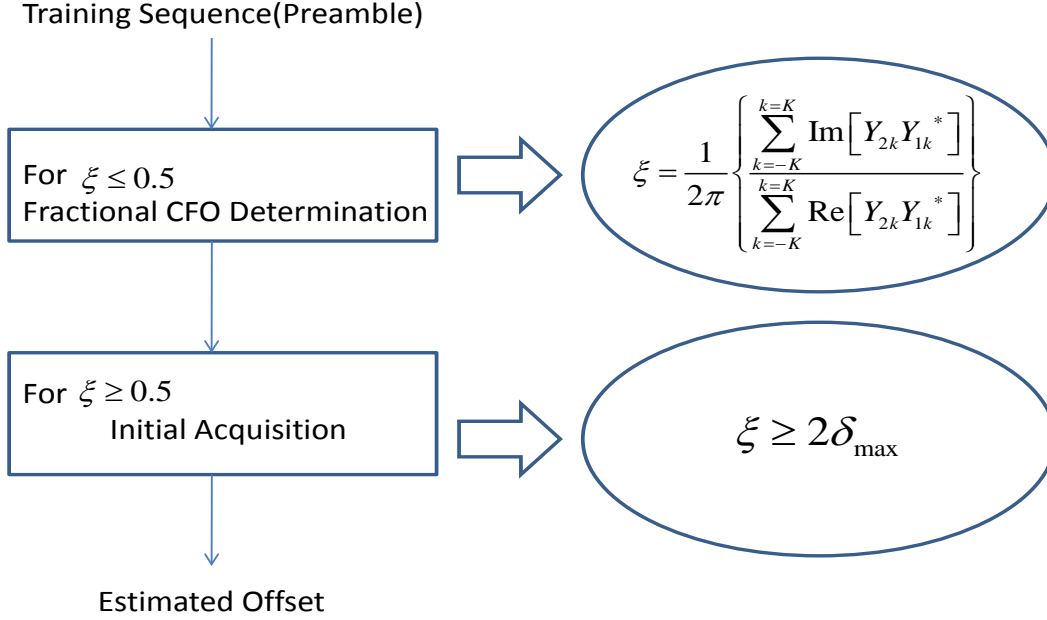


Fig. 5. MLE by Paul H. Moose

TABLE I  
PN SEQUENCE FOR TRAINING SYMBOLS [10]

freq	$c_{1,k}$	$c_{2,k}$	$v_k = \sqrt{2} \frac{c_{2,k}}{c_{1,k}}$
-4	$7+7j$	$5-5j$	$-j$
-3	0	$-5-5j$	
-2	$-7+7j$	$-5-5j$	$j$
-1	0	$-5+5j$	
0	$7+7j$	$-5-5j$	$-1$
1	0	$5+5j$	
2	$7-7j$	$-5+5j$	$-1$
3	0	$5-5j$	
4	$7+7j$	$5+5j$	$1$

to which the maximum correlation is obtained is declared as the best estimate [10].

After the two training symbols are frequency corrected by  $\frac{\hat{\phi}}{\pi T}$  (by multiplying the samples by  $e^{-j2t\frac{\hat{\phi}}{T}}$ ), let their FFTs be  $x_{1,k}$  and  $x_{2,k}$  and let the differentially-modulated PN sequence on the even frequencies of the second training symbol be  $v_k$ . The PN sequence will appear at the output except it will be shifted by  $2z$  positions because of the uncompensated frequency shift of  $\frac{2z}{T}$  [10]. Note that because there is a guard interval and there is still a frequency offset, even if there were no differential modulation between training symbols 1 and 2 (i.e., PN sequence, as given in table I), there would still be a phase shift. Since at this point the integer is unknown, this additional phase shift is unknown. However, since the phase shift is the same for each pair of frequencies, a metric similar to MLE can be used.

Let  $X$  be the set of indices for the even frequency compo-

nents

$$X = \{-W, -W + 2, \dots, -4, -2, 2, 4, \dots, W - 2, W\}$$

The number of even positions shifted can be calculated by finding  $\hat{g}$  [10], to maximize

$$B(g) = \frac{\|\sum_{k \in X} x_{1,k+2g}^* \cdot v_k^* \cdot x_{2,k+2g}\|^2}{2\{\sum_{k \in X} \|x_{2,k}\|^2\}^2} \quad (18)$$

with integer  $g$  spanning the range of possible frequency offsets and  $W$  being the number of even frequencies with the PN sequence. Then the frequency offset estimate would be

$$\hat{\Delta f} = \frac{\hat{\phi}}{\pi T} + \frac{2\hat{g}}{T} \quad (19)$$

## V. SIMULATION & RESULTS

The plots of bit energy/noise power versus bit error rate are given in figs. 6 and 7, for both [9] and [10]. These results have been generated by transmitting 10,000 bits and introducing AWGN and frequency selective fading, as a channel affect. The plots include a curve that shows the bit error rate for a system with compensation of CFO.

The given plots clearly indicate a reduction in the bit error rate for the increasing values of bit energy/noise power with the application of algorithms presented in [9] and [10].

With an increase in the bit energy /noise power ratio the variance of noise goes on decreasing, resulting in the corresponding decrease in the standard deviation of noise. With the decreasing values of standard deviation of noise and application of moose's algorithm [9], the bit errors in the received data go on decreasing, this is primarily due to the compensation of the CFO provided by the algorithm.

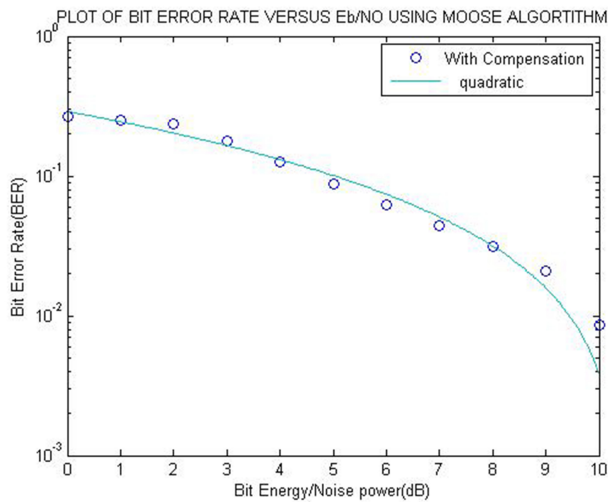


Fig. 6. BER vs Eb/No using Moose Algorithm

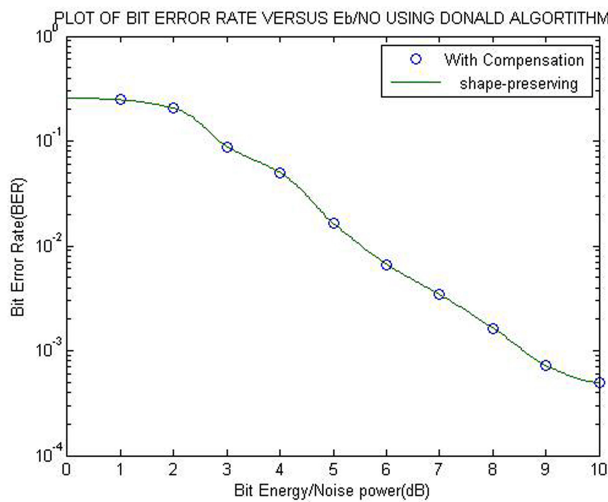


Fig. 7. BER vs Eb/No using Donalds Algorithm

It has been observed that the reduction in the bit error rate obtained with the Donalds algorithm [10] (fig 7) is more than that obtained by the Mooses algorithm [9] (fig 6). The authors in [9] estimated the fractional part of CFO by correlating the first part of the transmitted preamble with the second part and calculating the phase of that correlation, however this was done in frequency domain assuming that the timing detection has been performed. The authors in [10] estimated the fractional part of CFO by using the phase of the correlation of the first part of the preamble with the second part, however this was done in time domain corresponding to the best timing point.

Moreover, the estimates of the fractional parts of CFO by [9] and [10] match when the CFO is limited to 0.5. When CFO is greater than 0.5, [9] uses initial acquisition to increase the range of CFO estimation by using shorter training symbols, for example shortening the training symbols by a factor of two would double the acquisition range. The authors in [10] claim that this approach would work to a point and the estimates would get worse as the symbols get shorter because there are

fewer samples over which to average. Moreover, the training symbols need to be kept longer than the guard interval so that the channel impulse response does not cause distortion when estimating the frequency offset. So the authors in [10] proposed a separate method for the estimation of CFO, when it was greater than 0.5. The estimation of the integer part of CFO by the authors of [10] provides their algorithm an edge over the algorithm of [9], resulting in a more reduction in bit error rate as compared to [9].

## VI. CONCLUSION

This paper presented the concept and significance of frequency synchronization for 802.11n based MIMO-OFDM systems. Two types of synchronization issues i.e Carrier Frequency Offset (CFO) and Sampling Frequency Offset are highlighted. Moreover this paper also builds on the concept of CFO by comparing and analyzing the performance of two algorithms for Carrier Frequency offset correction. It has also been proved that the reduction in the bit error rate obtained with one of the algorithms, that estimates the integer part of CFO is more than that obtained by the algorithm that does not estimate the integer part of CFO. Thus, the estimation of integer part of the CFO is as important as estimating the fractional part, for nominal frequency synchronization.

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