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The Stability of Some Viable Stars and Electromagnetic Field

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We examine the impact of electromagnetic field on the stability of compact stars corresponding to embedded class one metric using the concept of cracking. For this purpose, we develop the generalized hydrostatic equilibrium equation for charged perfect fluid distribution of compact stars and perturb it by means of local density perturbation scheme to check the stability of inner matter configuration. We investigate the cracking of Her X-1, PSR 1937+21, PSR J 1614-2230, PSR J 0348+0432 and RX J 1856-37. We conclude that PSR J 0348+0432 and RX J 1856-37 exhibit cracking when charge is introduced on these astrophysical objects.

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The idea of embedding of four-dimensional space-time into larger dimensional space is an old and replicated one.[1−4] A few years ago, due to renewal in the interest of embedding symmetries, Pavsic *et al.*[5] presented many applications of embedding to general relativity (GR). Gupta *et al.*[6] presented new models of a charged super-dense star belong to embedding class one. Compact objects (CO) like white dwarfs, pulsars, neutron stars (NS), magnetars, belong to a significant class of those relativistic objects which were widely investigated in astrophysical research. In 1964 Bondi[7] proposed that the hydrostatic equilibrium equation can be used to test the stability of spherical CO. Herrera *et al.*[8,9] introduced the term of 'cracking' to explain the behavior of spherical symmetries. They further investigated how transverse cracking of CO can be induced by axially symmetric perturbations.[10] Prisco *et al.*[11,12] redefined the conditions of cracking for homogeneous CO. Abreu *et al.*[13] enhanced the concept of cracking by means of sound speed velocities. Gonzalez *et al.*[14,15] developed the scheme of local density perturbation (LDP) for further extension of the idea of cracking. Azam *et al.*[16,17] applied the concept of LDP to check the stability of charged and neutral CO in linear regime. They also implemented the LDP on a particular CO in quadratic regime.[18] Recently, Herrera *et al.*[19] investigated the cracking of a polytropic sphere through small fluctuations of local anisotropy.

Analyzing the consequences of charge on the natural properties of stars is an important subject in GR. In this scenario, Bonnor[20,21] investigated the effect of electromagnetic field (EF) on spherical CO and concluded that electric repulsion can halt the gravitational collapse. Bekenstein[22] studied the gravitational collapse of charged CO through generalized hydrostatic equilibrium equation. Ray *et al.*[23] calculated the approximated value of charge, (10^{20} C), required for a charged CO to stay in the equilibrium state. For instance, Her X-1 belongs to the class of massive x-ray binaries with mass approximately $2M_{\odot}$, which lies between low and high mass x-ray binaries. Due to its intermediate mass such a system is very rare where one component is either an NS or black hole.[24,25] Pulsar PSR 1937+21 has been studied by many researchers to describe its properties like mass, radii, rotation and radio pulse.[26−29] PSR J 1614-2230 is NS with a white dwarf in its binary system having mass $1.97 \pm 0.04M_{\odot}$. Demorest *et al.*[30] analyzed CO by means of Shapiro delay and presented the observed values of different physical parameters for PSR J 1614-2230. Tauris *et al.*[31] presented the mathematical model for PSR J 1614-2230 to show that PSR J 1614-2230 was born as massive NS. PSR J 0348+0432 was discovered in 2007 with the Green Bank Telescope in a drift-scan survey and it has mass $2.01 \pm 0.04M_{\odot}$. The mass of PSR J 0348+0432 is slightly greater than the mass of PSR J 1614-2230, while both are statistically indistinguishable stars.[32] The NS RX J 185637 was recently detected by the Hubble Space Telescope and its mass is about $0.9041M_{\odot}$. The radius of RX J 185637 is approximately 6 km and its distance from earth is around 200 light years.[33]

The magnetars belong to a family of NS with an extremely powerful magnetic field. In 2012 Gao *et al.*[34] presented a mechanism for magnetar soft x-ray or γ-ray emission. They *et al.*[34−37] discussed several aspect of magnetars like pressure of electrons, equation of state and constraints of braking indices in the presence of superhigh magnetic fields. In this work, we apply the technique of cracking on some charged spherically symmetric CO corresponding to embedding class one metric presented by Maurya *et al.*[38]

We consider the line element for a static spherically symmetric space time in curvature coordinates given by

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1}$$

The above metric corresponds to the embedding class one, if the Karmarkar[39] condition is satisfied, i.e., $R_{1414} = \frac{R_{1212}R_{3434}+R_{1224}R_{1334}}{R_{2323}}$, with $R_{2323} \neq 0$.[40] For spherically symmetric metric Eq. (1), the Einstein–Maxwell field equations are given by

$$\frac{\nu' e^{-\lambda}}{r} - \frac{(1-e^{-\lambda})}{r^2} = 8\pi P - E^2, \tag{2}$$

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$$e^{-\lambda}\Big[\frac{\nu''}{2}-\frac{\nu'\lambda'}{4}+\frac{\nu'^2}{4}+\frac{\lambda' r-\nu'}{2r}\Big]=8\pi P+E^2, \quad (3)$$

$$\frac{\lambda' e^{-\lambda}}{r}+\frac{(1-e^{-\lambda})}{r^2}=8\pi\rho+E^2, \quad (4)$$

where ρ is the energy density, E is the EF, and P is pressure of the fluid. Solving Eqs. (2)–(4) simultaneously, we obtain the hydrostatic equilibrium equation for CPF

$$\frac{dP_r}{dr}=-\frac{1}{2}(\rho+P)\nu'+\frac{E}{4\pi r^2}(r^2E)', \quad (5)$$

which shows that the gradient of pressure is effected by EF. Using the relations $e^{\nu}=1-2m/r+q^2/r^2$ and $q=Er^2$ given in Ref. [18], Eq. (5) yields

$$\begin{aligned}\varOmega\equiv{}&-\frac{dP}{dr}+(\rho+P)\frac{(-2m-8\pi r^3P+2r^3E^2)}{2r^2-4mr+2r^4E^2}\\&+\frac{(r^2E)'E}{4\pi r^2}=0.\end{aligned} \quad (6)$$

We will use Eq. (6) to investigate the stability of charged CO through LDP in the presence of EF.

Table 1. Cracking of CO in the presence of EF.

Star	$A(10^{-13})$	B	D	$K(10^{12})$	R (km)	$E>0$
Her X-1	3.6319	0.5552	3.0626	3.7972	6.700	Stable
PSR 1937+21	1.9503	0.4103	2.5857	8.0775	11.4998	Stable
PSR J1614-2230	1.8112	0.4419	2.6998	8.4338	11.3664	Stable
PSR J0348-0432	1.8017	0.4228	2.6315	8.6369	11.7372	Unstable at $R_{\rm c}=10.2103$ km
RX J1856-37	5.0962	0.5189	2.9540	2.7925	6.006	Unstable at $R_{\rm c}=5.8058$ km

For this, we perturb the static configuration of CO filled with CPF through LDP. Here $\varOmega$ represents the state of hydrostatic equilibrium of the system. Thus to check the stability of the CO, we should observe the behavior of $\varOmega$ just after equilibrium is perturbed by LDP. This perturbed state of CO is represented by $\delta\varOmega$ and any change in its state can be observed numerically by testing change in its sign.[9,17] The overturning or cracking will occur in the inner region of CO when the equilibrium state is altered due to reversal in the direction of perturbed force, i.e., $\delta\varOmega>0\rightarrow\delta\varOmega<0$ and vice-versa.[17] We apply the LDP to all the physical quantities and their derivatives involved in Eq. (6) as $\frac{dP}{dr}(\rho+\delta\rho)$, $P(\rho+\delta\rho)$, $m(\rho+\delta\rho)$, $E'(\rho+\delta\rho)$, and $E(\rho+\delta\rho)$. The perturbed form of Eq. (6) is given by

$$\varOmega=\varOmega_0(\rho,P,P',m,E,E')+\delta\varOmega,$$

where

$$\begin{aligned}\frac{\delta\varOmega}{\delta\rho}={}&\frac{\partial\varOmega}{\partial\rho}+\frac{\partial\varOmega}{\partial P}v_r^2+\frac{\partial\varOmega}{\partial P_r'}(v_r^{2\prime}+v_r^2\rho''(\rho')^{-1})\\&+\frac{4\pi r^2}{\rho'}\frac{\partial\varOmega}{\partial m}(\rho+\frac{E^2}{2})+\frac{\partial\varOmega}{\partial E}\frac{E'}{\rho'}+\frac{\partial\varOmega}{\partial E'}\frac{E''}{\rho'}.\end{aligned} \quad (7)$$

This is the basic equation used to describe the effect of LDP on the cracking of CPF inner matter distribution of CO corresponding to embedding class one with $v_r^2=\frac{dP}{d\rho}$ as the radial sound speed. We will plot $\frac{\delta\varOmega}{\delta\rho}$ against radius 'r' of the star for different values of the parameters involved in the model. Using Eq. (6), the derivatives involved in Eq. (7) are given by

$$\begin{aligned}\frac{\partial\varOmega}{\partial\rho}&=\frac{-2m-8\pi r^3P+2r^3E^2}{2r^2-4mr+2r^4E^2},\\\frac{\partial\varOmega}{\partial m}&=\frac{(\rho+P)(-4r^2-32\pi r^4P+4r^4E^2)}{(2r^2-4mr+2r^4E^2)^2},\\\frac{\partial\varOmega}{\partial P}&=\frac{-2m-16\pi r^3P-8\pi r^3\rho2r^3E^2}{2r^2-4mr+2r^4E^2},\\\frac{\partial\varOmega}{\partial E'}&=\frac{E}{8\pi},\quad\frac{\partial\varOmega}{\partial P'}=-1,\\\frac{\partial\varOmega}{\partial E}&=\frac{(\rho+P_r)(8r^5EE'-16mr^4EE'+4r^7EE')}{(2r^2-4mr+2r^4E^2)^2}\\&\quad+\frac{4EE'+rE'}{4\pi r}\\&\quad+\frac{(8mr^4EE'+32\pi r^7PEE'-4r^7E^3E')}{(2r^2-4mr+2r^4E^2)^2}.\end{aligned}$$

The model presented by Maurya *et al.*[38] is defined by

$$\frac{2m(r)}{Ar^3}=\frac{De^{2Ar^2}}{ADr^2e^{2Ar^2}+1}+\frac{Ar^2(e^{4Ar^2}D^2-4e^{2Ar^2}D+4)}{2(ADr^2e^{2Ar^2}+1)^2}, \quad (8)$$

$$\rho=\frac{A(6De^{2Ar^2}(2Ar^2+1)-4Ar^2+AD^2r^2e^{4Ar^2})}{16\pi(ADr^2e^{2Ar^2}+1)^2}, \quad (9)$$

$$P=\frac{A(4Ar^2+2De^{2Ar^2}(2Ar^2-1)-AD^2r^2e^{4Ar^2}+8)}{16\pi(ADr^2e^{2Ar^2}+1)^2}, \quad (10)$$

$$E^2=\frac{A^2r^2(e^{2Ar^2}D^2-4e^{2Ar^2}D+4)}{2(ADr^2e^{2Ar^2}+1)^2}. \quad (11)$$

The expressions for the radial sound speed velocity[13] can be obtained from Eqs. (9) and (10) as

$$\begin{aligned}v_r^2={}&(16De^{2Ar^2}-3D^2e^{4Ar^2}+8A^2D^2r^4e^{4Ar^2}\\&+28ADr^2e^{2Ar^2}+4AD^2r^2e^{4Ar^2}\\&+16A^2Dr^4e^{2Ar^2}-AD^3r^2e^{6Ar^2}\\&-4)/(11D^2e^{4Ar^2}-24De^{2Ar^2}\\&+24A^2D^2r^4e^{4Ar^2}-28ADr^2e^{2Ar^2}\\&+20AD^2r^2e^{4Ar^2}-16A^2Dr^4e^{2Ar^2}\\&+AD^3r^2e^{6Ar^2}+4),\end{aligned}$$

where A, B, K and $D=4ABK$ are the parameters involved in the model of Maurya *et al.*[38] The values of these parameters for CO under consideration are listed in Table 1.

Figures 1 and 2 show that all CO under consideration are stable in the absence of EF ($E = 0$). Recently, we[17,18] have studied the cracking of PSR J1614-2230 through LDP analysis in the scenario of linear and quadratic regimes with and without EF corresponding to the model of Takisa *et al.*[41] and found that PSR J1614-2230 remains unstable in each regime. Thus the model of Maurya *et al.*[38] is a physically viable candidate for neutral CO corresponding to embedded class one metric, and we can summarize that PSR J 1614-2230 remains unstable in the linear and quadratic regime corresponding to the model of Takisa *et al.*[42] for anisotropic fluid, while it remains stable corresponding to the model of Maurya *et al.*[38] for perfect fluid. However, from Fig. 2, it is clear that PSR J 0348+0432 and RX J 1856-37 exhibit cracking (overturning) in the presence of EF.

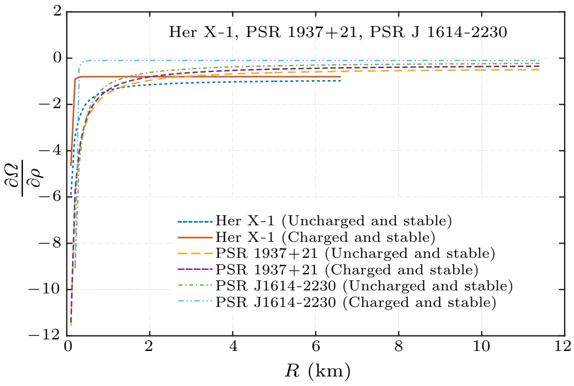

Fig. 1. Plots show that the curves do not change their signs with both $E > 0$ and $E = 0$, i.e., Her X-1, PSR 1937+21, PSR J 1642-2230 remain stable for different values of the parameters involved in the model.[38]

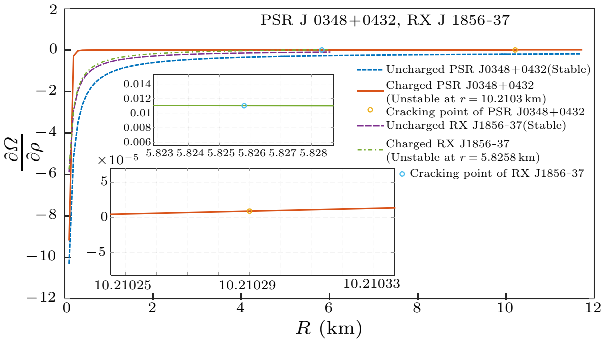

Fig. 2. Plots show that cracking points occur at $R_c = 10.2103$ km and $R_c = 5.8258$ km for PSR J 0348+0432 and RX J 41856-37, respectively. They become potentially unstable for $E > 0$, while stable for $E = 0$.

We have used the model of Maurya *et al.*[38] to examine the cracking of CO Her X-1, PSR 1937+21, PSR J 1614-2230, PSR J 0348+0432 and RX J 1856-37 with and without charge, respectively. Figure 1 shows the stability of celestial objects Her X-1, PSR 1937+21 and PSR J 1614-2230 for different values of parameters[38] listed in Table 1. It is observed that Her X-1, PSR 1937+21 and PSR J 1614-2230 remain stable in the presence and absence of EF corresponding to embedded class one. Figure 2 indicates that PSR J 0348+0432 and RX J 1856-37 exhibit cracking with the inclusion of charge. The values of cracking point for PSR J 0348+0432 is $R_c = 10.2103$ km, whereas for RX J 1856-37 cracking occurs at $R_c = 5.8058$ km. It is noted that the LDP technique does not effect the stability of CO in the neutral case, but drastically changes the stability with the inclusion of charge in the framework of GR with embedded class one. The LDP can be applied to magnetars to check the stability under the influence of strong magnetic field. The models and discussion presented by Gao *et al.*[34−37] can be used for implementation of LDP to check the cracking of magnetars, while it cannot be included in this work due to special features of magnetars. We conclude that Her X-1, PSR 1937+21 and PSR J 1614-2230 remain stable, while PSR J 0348+0432 and RX J 1856-37 may become potentially unstable in the framework of GR.

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