Existence results for a system of fractional differential equations subject to coupled integral boundary conditions

by

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Abstract

Recently, Johnny Henderson and Rodica Luca [1], have presented some new existence and uniqueness results, and for this, they have used a variety of theorems. They have worked on fractional di erential equations, and have investigated the uniqueness and existence for non-negative solutions of a system of nonlinear Riemann-Liouville fractional di erential equations

$$D_{0+}v_1(t) + {}_1f(t; v_1(t); v_2(t)) = 0; \quad \text{where } t \in (0; 1) \text{ and } n-1 < 6\, n$$

$$D_{0+}v_2(t) + {}_2g(t; v_1(t); v_2(t)) = 0; \quad \text{where } t \in (0; 1); \text{ and } m-1 < 6\, m$$

with the coupled integral boundary conditions

$$v_1(0) = v_1'(0) = \cdots = v_1^{(n-2)}(0) = 0; \quad v_1(1) = \int_0^1 v_2(s)dH_1(s);$$

$$v_2(0) = v_2'(0) = \cdots = v_2^{(m-2)}(0) = 0; \quad v_2(1) = \int_0^1 v_1(s)dH_2(s);$$

where $m; n \in \mathbb{N}$; $m; n > 3$; D_{0+} and D_{0+} are the derivatives from Riemann-Liouville with orders , respectively. Further, the integrals in the boundary conditions are the integrals from Riemann-Stieltjes. Some adequate conditions will be given on the parameters ${}_1$, ${}_2$, and nonlinear functions f and g, so that non-negative solutions of the above problem exist. This thesis is detailed review of the results presented in [1].

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