

GENERAL SOLUTIONS FOR UNSTEADY FREE
CONVECTIVE FLOW FOR BRINKMAN TYPE FLUID

By
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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled “**General solutions for unsteady free convective flow for Brinkman type fluid**” by **Maryam Aleem** in partial fulfillment of the requirements for the degree of **Master of Philosophy**.

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Abstract

The main aspire of this desertion is to represent generalized solutions for Brinkman type non-Newtonian fluids in which we will reckon time dependent velocity along the boundary.

In chapter 1, a few cardinal concepts concerning the rates of flows, dissimilar types of fluids, a couple of constitutive equations and equations of apparent movement , Navier Stokes equations and integral transform i.e., The Laplace transform are dis-coursed.

In chapter 2 , an analysis is followed by the un-firm or jerky free convection stream of a Brinkman type fluid flowing across an upright plate subject to time dependent velocity $f(t)$ which fulfills the condition of $f(0)=0$. By inserting the suitable variables, the basic governing equations are abbreviated to dimensionless equations sound to all levied initial and boundary conditions. The exact solutions of initial value problem has been attained by using Laplace transform technique. When $\beta^* = 0$, the results will be true for Newtonian fluid for the analogous motion. The impression of assorted values of the tangible parameters specified for temperature, skin friction, velocity and Nusselt number is underscored diagrammatically.

In chapter 3, the velocity domain of jerky convection stream of Brinkman type flowing materials passing through an upright plate imbedded in permeable medium with mass diffusion and Newtonian heating condition is dissected. The exact solution is received by means of Laplace transform and deduced to the results for Newtonian fluids. The effects of several parameters upon the velocity dispersion, temperature, mass diffusion and skin friction is tested graphically.

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Nomenclature

\mathbf{V}	Velocity field
\mathbf{L}	Velocity gradient
\mathbf{a}	Acceleration
ρ	Fluid mass density
$\nabla \cdot \mathbf{V}$	Divergence of vector field \mathbf{V}
p	Pressure of the fluid
\mathbf{I}	Identity tensor
\mathbf{T}	Cauchy stress tensor
\mathbf{S}	Extra stress tensor
μ	Dynamic viscosity
\mathbf{A}_1	First Rivlin-Ericksen tensor
$\nabla \mathbf{V}$	Gradient of the vector field \mathbf{V}
\mathbf{A}_2	Second Rivlin-Ericksen tensor
q	Laplace transform parameter
$\nu = \frac{\mu}{\rho}$	Kinematic viscosity of the fluid
β	Slip coefficient
$H(t)$	Heaviside unit step function
k	Permeability of the porous medium
ϕ	porosity
γ	Ratio of specific Heat
Sc	Schmidt number