Third-Order Numerical Method for the Solution of Heat Equation with Nonlocal Boundary Conditions

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بسم الله الرحمن الرحيم

In the name of Allah

the most Gracious

the most Compassionate

Contents

List of Tables

List of Figures

Abstract

A third order numerical technique is developed for solving one dimensional non-homogenous heat equation with integral boundary conditions. In this method second order spatial derivative is approximated by third order nite di erence approximation. The parallel splitting technique combined with Simpson's 1/3 rule is used to tackle this problem. This method is very precise due to highly accurate results.

Chapter 1

Introduction

In this chapter, di rent types of di erential equations and nite di erence meth-ods will be presented.

1.1 Di erential Equation

Mathematical models are used to tackle real world problems which can be trans-form into mathematical equations. The motion of a particle in a straight line, the motion of a missile, the function of a nuclear reactor, transfer of heat, vi-bration of a particle and chemical reaction etc. are governed by di erential equations along with initial conditions and boundary conditions which may be local or non-local boundary conditions. Di erential equations have one or more than one terms in which derivatives of dependent variable with respect to one or more independent variables involved e.g.

$$\frac{dy}{dx} + y\cos x = \sin x;$$

$$\frac{d^2y}{dx^2} + xy\left[\frac{dy}{dx}\right]^2 = 0;$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2};$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2};$$