

**Third-Order Numerical Method for
the Solution of Heat Equation with Nonlocal Boundary
Conditions**



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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

*In the name of Allah
the most Gracious
the most Compassionate*

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Abstract

A third order numerical technique is developed for solving one dimensional non-homogenous heat equation with integral boundary conditions. In this method second order spatial derivative is approximated by third order finite difference approximation. The parallel splitting technique combined with Simpson's 1/3 rule is used to tackle this problem. This method is very precise due to highly accurate results.

Chapter 1

Introduction

In this chapter, different types of differential equations and finite difference methods will be presented.

1.1 Differential Equation

Mathematical models are used to tackle real world problems which can be transformed into mathematical equations. The motion of a particle in a straight line, the motion of a missile, the function of a nuclear reactor, transfer of heat, vibration of a particle and chemical reaction etc. are governed by differential equations along with initial conditions and boundary conditions which may be local or non-local boundary conditions. Differential equations have one or more than one terms in which derivatives of dependent variable with respect to one or more independent variables involved e.g.

$$x \frac{dy}{dx} + y \cos x = \sin x;$$

$$x^2 \frac{d^2y}{dx^2} + xy \left[\frac{dy}{dx} \right]^2 = 0;$$

$$\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = \frac{d^2y}{dx^2};$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2};$$