

On Topological Properties Of The Line Graphs Of Certain Chemical Structures



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First and foremost, I would like to thank **Allah Almighty**, the Supreme power. With-out His will, nothing is possible. May Allah shower His blessings upon His messenger **Hazrat Muhammad (SAW)** who received the first revelation IQRA, meaning Read or Recite.

I would like to thank my supervisor **Dr. Sohail Zafar**, Assistant Professor, University of Management and Technology Lahore, his always exuding positive energy never let me down during my research work. He always responded to my feedback and suggestions and guided me with great aplomb and grace. He enabled me to analyze mathematical problems from a multi dimensional vision. I greatly benefitted from his keen insight for solving seemingly intractable difficulties and put them into simple terms. I have great respect for his humble attitude. I am fortunate to have the chance to work with him. I would also like to thank my co-supervisor **Dr. Zohaib Zahid**, Assistant Professor, University of Management and Technology Lahore, for his great abilities in handling mathematical research tools and support my work to run smoothly. I am also thankful for his advice and his willingness to share his bright thoughts with me, for shaping up my research.

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RAHEELA SOHAIL

DECLARATION

I **Raheela Sohail**w/o**Sohail Bashir**ID: **15008109003**.Session 2015– 2017 hereby declare that the matter printed in the thesis titled “**On Topological Properties Of The Line Graphs Of Certain Chemical Structures**” is my own work and has not been printed, published and submitted as research work, thesis or publication in any form in any University, Research institution etc. in Pakistan or Abroad.

Dated: February, 15, 2018

Abstract

Let $G = (V(G); E(G))$ be a simple connected graph, where $V(G)$ is the set of vertices and $E(G)$ is the set of edges. A topological index of a graph is a number related to a graph which is invariant under graph automorphisms. In this thesis, we give theoretical results for some topological indices such as geometric-arithmetic index $GA(G)$, fifth geometric-arithmetic index $GA_5(G)$, atom-bond connectivity index $ABC(G)$, fourth atom-bond connectivity index $ABC_4(G)$, Zagreb indices $M_1(G)$, $M_2(G)$, $M_3(G)$, $M_1(G)$, $M_2(G)$, Zagreb coindices $M_1(\overline{G})$, $M_2(\overline{G})$, $\overline{M_2(G)}$, Randic index $R(G)$, hyper-Zagreb index $HM(G)$, general sum-connectivity index $\chi(G)$ and augmented Zagreb index $AZI(G)$ by considering graph G as a line graph of V phenylenic nanotube, V phenylenic nanotorus, linear pentacene and multiple pentacene.

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RAHILA SOHAIL

Table of Contents

Abstract	1
Acknowledgements	2
Table of Contents	1
Introduction	3
1 Basic notions	7
1.1 Fundamentals	7
1.2 Types of graph	8
1.3 Compliment of a graph	11
1.4 Line graph of a graph	11
1.5 Graphs of different types of phenylenic and pentacenes nanostructures	12
1.5.1 Graph of V -phenylenic nanotube	12
1.5.2 Graph of V -phenylenic nanotorus	13
1.5.3 Graph of linear [n]-pantacene	14
1.5.4 Graph of mutiple pantacene	15
2 Topological indices(Degree-based)	17
2.1 First, second and third Zagreb indices	17
2.2 Hyper Zagreb index	18

2.3	Zagreb co-indices	19
2.4	Randic index	19
2.5	Atom-bond connectivity index	19
2.6	The general sum-connectivity index	20
2.7	First geometric arithmetic index	20
2.8	Augmented Zagreb index	20
2.9	Fourth atom-bond connectivity index	21
2.10	Fifth geometric-arithmetic index	21
3	Line graphs of V -phenylenic nanostructures and their topological indices	23
3.1	Line graphs of V -phenylenic nanostructures	23
3.1.1	Line graph of V -phenylenic nanotube	24
3.1.2	Line graph of V -phenylenic nanotorus	24
3.2	The edge partition of the line graphs of V -phenylenic nanostructures w.r.t degrees	25
3.3	The edge partitions of line graphs of V -phenylenic nanostructures w.r.t degree sum	26
3.4	Topological indices of line graph of V -phenylenic nanotube and V -phenylenic nanotorus	27
4	Line graphs of certain pentacenes based on topological indices	32
4.1	Line graphs of certain pentacenes	32
4.1.1	Line graph of linear pantacene	32
4.1.2	Line graph of multiple pantacene	33
4.1.3	The partition of line graph G_3 and G_4 of linear pantacene and multiple pantacene	34
4.2	The edge partitions of line graphs of linear pantacene and multiple pantacene	35
4.3	Topological indices of line graph of linear pantacene and multiple pantacene	36
	Bibliography	41

Introduction

Euler (1701-1783) is considered to be the father of graph theory with his work towards the solution of the problems in geometry (see [27, 34, 35]). A graph is a representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by vertices (also called nodes or points) and the set of vertices is denoted by $V(G)$ and is called vertices set. The links that connect some pairs of vertices are called edges (also called arcs or lines) and the set of edges is denoted by $E(G)$ and is called edges set. Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$.