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Strain Energy Based Homogenization Method to Find The Equivalent Orthotropic Properties of Sandwich Structures

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Abstract: The purpose of present study is to present a methodology to determine the effective elastic properties of sandwich structures. This methodology is based on strain energy based criteria. Sandwich structures contain core material and face sheet. Earlier work in this domain contains the homogenization of core material and determining its equivalent orthotropic properties. The equivalent properties for core material then modeled along with face sheet for final analysis. In the present study however a direct scheme is proposed. Here core material and face sheet are modeled together to determine the equivalent orthotropic properties of sandwich structure.

Keywords: Homogenization, Sandwich structures, Finite Element Analysis, Orthotropic

1. INTRODUCTION

Composite sandwich structures have been widely used in aerospace structures, ship building, infrastructure, etc. due to their light weight and high strength to weight ratio. Traditionally, light-weight core materials such as foam core, truss core, honeycomb core, aluminum core have been used in fabricating sandwich structures. A typical sandwich structure consists of honeycomb core material covered by face sheets on both sides.

The purpose of the present study is to present a strain energy based homogenization method to determine the equivalent elastic orthotropic properties of whole sandwich panel including honeycomb core and face sheets as well. The required sandwich panel properties are the two in-plane Young's moduli (E_{11}, E_{22}), the out-of-plane Young's modulus (E_{33}), the in-plane shear modulus (G_{12}), the out of-plane shear moduli (G_{13}, G_{23}), and the three Poisson ratios. (υ_{12}, υ_{13}, υ_{23}). This method assumes the mechanical equivalence between the microstructures of RVE (Representative Volume Element) and a similar homogeneous macroscopic volume.

Previously, main work focuses on determination of equivalent properties for honeycomb core materials without face sheets. Different authors developed analytical tools to determine homogenized properties. Kelsey *et al.* developed a methodology to determine the out-of-plane shear moduli (G_{13}, G_{23}) for honeycomb core materials. Based on Kelsey *et al.*, Gibson and Ashbay derived all nine orthotropic properties for honeycomb materials with constant wall thickness. This approach is further modified by Zhang and Ashbay to include the double wall thickness for the out of plane values. The theory of Ashbay and Gibson was also modified by Klintworth and Stronge adding the influence of the double thickness walls to the in-plane shear modulus. Hohe and Becker and Yang and Becker published the work regarding the cellular sandwich cores considering homogenization techniques.

In recent times finite element analysis has become a popular tool to determine the elastic properties of honeycomb materials. Gornet *et al.* developed a 3D finite element based model to determine the elastic properties of core materials using periodic boundary conditions Zhang and Xia. In the present study the same approach used by Gornet *et al.* was adopted to determine the equivalent properties of sandwich structures considering both honeycomb and face sheet material. The original code was written to find the properties of honeycomb core material only at Ecole Centrale de Nantes, France. The same code is now further modified for sandwich structures, modeling the effect of both honeycomb core and face sheets, hence directly determining the properties of sandwich structure as a whole. This paper is organized as follows. Modeling and simulation of homogenization for RVE is done in finite element software CAST3M.

Details of RVE for both single honeycomb core and sandwich panel are presented in section 1. In the section 2, homogenization method along with necessary equations is described. Section 3 details the 3D finite element simulations and results for single honeycomb core. 3D finite element simulations for

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sandwich structure along with results are presented in section 4 and are compared with available analytical techniques. Concluding remarks are given in section 5.

2. Representative Volume Element (RVE):

Representative volume element selected for over-expanded hexagonal honeycomb material is shown in **(Fig.1).** Axis 1, 2 and 3 represent the necessary orthotropic basis for the homogeneous element, Any larger volume of complete honeycomb material can be obtained by successive translation of RVE.

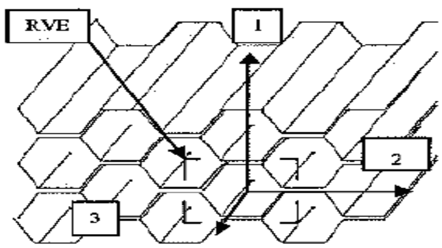

Fig. 1: Representative Volume Element (RVE)

Representative volume element for honeycomb core material and corresponding equivalent homogenized volume are shown in **(Fig.2)**. Similarly RVE for sandwich structure including both honeycomb core and face sheets and corresponding homogenized volume are shown in **(Fig.3)**. Equations and formulation adopted to determine the equivalent orthotropic properties of homogenized volume are given in next section. Once the orthotropic properties of homogenized volume are known, one can model the large and complex sandwich structures with simple homogenized volume for different finite element analysis.

3. MATERIAL AND METHODS

Homogenization method:

By considering Y the RVE and $|Y|$ its volume measurement, the space average of a second order tensor X_{IJ} (stress or strain) can be written of following form:

$$\langle X_{IJ} \rangle = \frac{1}{|Y|} \int_Y X_{IJ}(Y)\,dY \qquad (1)$$

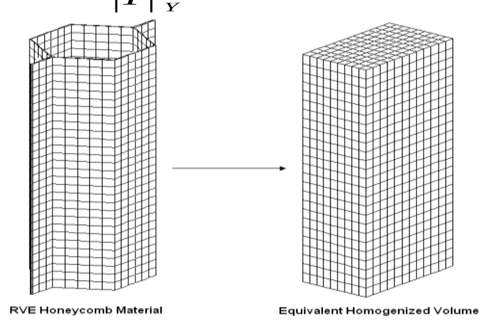

Fig. 2: Honeycomb core RVE

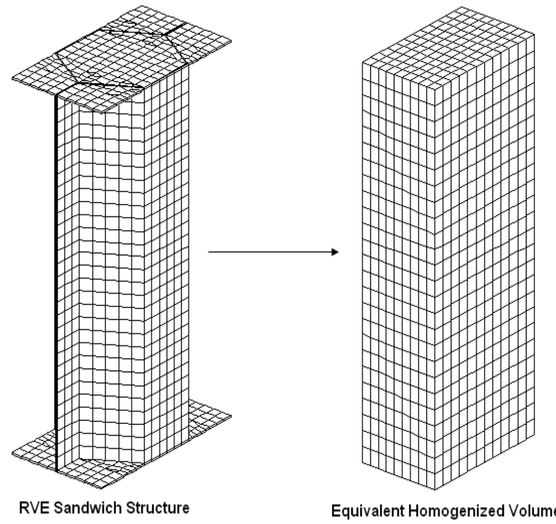

Fig.3: Sandwich structure RVE

The average stress and strain tensors are written as $\Sigma_{IJ} = \langle \sigma_{IJ} \rangle$ and $\mathrm{E}_{IJ} = \langle \varepsilon_{IJ} \rangle$ respectively. Periodic homogenization is based on the data average of the strain field and then one determines the compliance operator C^{RVE} of the RVE by imposing displacement boundary conditions. Here, Hill-Mandel, energy approach is used to calculate the equivalent homogenized components of compliance matrix, described by a six by six matrix C_{IJ}^{RVE}. The homogenized behavior can be written as:

$$\Sigma = C^{RVE} E \qquad E = S^{RVE} \Sigma \qquad (2)$$

Where S^{RVE} describes the six by six stiffness matrix. The homogenized mechanical stiffness characteristics (C^{RVE}) of the RVE can be derived with the Hill-Mandel theorem. This theorem states that the RVE energy is equal to the average of the energies of its components:

$$\varepsilon_I C_{IJ}^{RVE} \varepsilon_J = \frac{1}{|Y|^{RVE}} \int_Y Tr\left[\sigma\varepsilon\right] dY \qquad (3)$$

The 21 linear combinations of applied loadings on RVE permits to determine the compliance matrix C_{IJ}^{RVE}. Equations correspond to 21 linear combinations of applied loadings to determine the 21 components of orthotropic compliance matrix C_{IJ}^{RVE} are given below:

$$\varepsilon_I C_{II}^{RVE} \varepsilon_I = \frac{1}{|Y|^{RVE}} \int_Y Tr[\sigma\varepsilon] dY \qquad I \in [1,3] \qquad (4)$$

$$\sqrt{2}\varepsilon_I C_{II}^{RVE} \sqrt{2}\varepsilon_I = \frac{1}{|Y|^{RVE}} \int_Y Tr[\sigma\varepsilon] dY \qquad I \in [4,6] \qquad (5)$$

$$\varepsilon_I C_{II}^{RVE} \varepsilon_I + 2\varepsilon_I C_{IJ}^{RVE} \varepsilon_J + \varepsilon_J C_{JJ}^{RVE} \varepsilon_J = \frac{1}{|Y|^{RVE}} \int_Y Tr[\sigma\varepsilon] dY \qquad (I,J) \in [1,3]^2 \text{ with } (I \neq J) \qquad (6)$$

$$\varepsilon_I C_{II}^{RVE} \varepsilon_I + 2\varepsilon_I C_{IJ}^{RVE} \sqrt{2}\varepsilon_J + \sqrt{2}\varepsilon_J C_{JJ}^{RVE} \sqrt{2}\varepsilon_J = \frac{1}{|Y|^{RVE}} \int_Y Tr[\sigma\varepsilon] dY \qquad (I,J) \in [1,3]\times[4,6] \qquad (7)$$

$$\sqrt{2}\varepsilon_I C_{II}^{RVE} \sqrt{2}\varepsilon_I + 2\sqrt{2}\varepsilon_I C_{IJ}^{RVE} \sqrt{2}\varepsilon_J + \sqrt{2}\varepsilon_J C_{JJ}^{RVE} \sqrt{2}\varepsilon_J = \frac{1}{|Y|^{RVE}} \int_Y Tr[\sigma\varepsilon] dY \qquad (8)$$

$$\text{with} \qquad (I,J) \in [4,6]^2 \text{ with } (I \neq J)$$

Equations (4)-(8) are used to find the components of compliance matrix C_{IJ}^{RVE}. Once the components of compliance matrix C_{IJ}^{RVE} are known, one can find the components of stiffness matrix S_{IJ}^{RVE} using relation $S_{IJ}^{RVE} = \left[C_{IJ}^{RVE}\right]^{-1}$. Now, two in-plane Young's moduli (E_{11}, E_{22}), the out-of-plane Young's modulus (E_{33}), the in-plane shear modulus (G_{12}), the out of-plane shear moduli (G_{13}, G_{23}), and the three Poisson ratios (υ_{12}, υ_{13}, υ_{23}) can be determined from stiffness matrix S_{IJ}^{RVE}. Components of six by six C_{IJ}^{RVE} and S_{IJ}^{RVE} matrices are given below:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{13} \\ \sqrt{2}\sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \sqrt{2}\varepsilon_{23} \\ \sqrt{2}\varepsilon_{13} \\ \sqrt{2}\varepsilon_{12} \end{bmatrix} \qquad (9)$$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \sqrt{2}\varepsilon_{23} \\ \sqrt{2}\varepsilon_{13} \\ \sqrt{2}\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{13} \\ \sqrt{2}\sigma_{12} \end{bmatrix} \qquad (10)$$

The relation between components of stiffness matrix and required nine orthotropic elastic components are below:

$$\begin{aligned} &S_{11} = \frac{1}{E_{11}}, S_{12} = -\frac{\upsilon_{12}}{E_{11}}, S_{13} = -\frac{\upsilon_{13}}{E_{11}} \\ &S_{21} = -\frac{\upsilon_{12}}{E_{22}}, S_{22} = \frac{1}{E_{22}}, S_{23} = -\frac{\upsilon_{23}}{E_{22}} \\ &S_{31} = -\frac{\upsilon_{31}}{E_{33}}, S_{32} = -\frac{\upsilon_{31}}{E_{33}}, S_{33} = \frac{1}{E_{33}} \\ &S_{44} = \frac{1}{2G_{23}}, S_{55} = \frac{1}{2G_{13}}, S_{66} = \frac{1}{2G_{12}} \end{aligned} \qquad (11)$$

4. Finite Element simulations and results:

CAST3M is a computer code for analysis of structures with the help of finite element method. This code was developed by the "Département Mécanique *et al.*, Technologie (DMT) du Commissariat français à l' Energie Atomique (CEA), Frnace". The purpose of the development of CAST3M is to provide high level support for the design and analysis of structures and components in the research field. In this context, CAST3M incorporates not only the process of calculations but also serves as pre-processor for the modeling of structural parts and analysis of the final results (post-treatment).

Honeycomb core material and face sheets are modeled with three dimensional brick elements having 20 nodes (CU20). Material for honeycomb core and face sheets is aluminum alloy. The core thickness is 25 mm with 1 mm aluminum alloy facing sheets. The core refers to a hexagonal honeycomb with density of 83.3 Kg/m^3, a cell size of 6.35 mm, which is made of aluminum alloy sheet with thickness of 0.0635 mm. Finite element analysis for the honeycomb material is done for 21 different loading conditions to determine the 21 orthotropic coefficients of compliance matrix. The principal orthotropic directions are shown in Fig.1. Finite element results of core material for three tensile loading conditions are shown in **(Fig. 4. 5)**. The determined orthotropic properties and its comparison are given in **(Table 1).**

In **(Table 1)**, results obtained by FE analysis are compared with methods proposed by different authors. Honeycomb core are usually designed for out of plane normal loading (E11) and out of plane shear loading (G13, G23) conditions. Ashbay and Gibson presented the solution for orthotropic properties for honeycomb cores. But formulation proposed by Ashbay and Gibson does not include the effects of double wall thickness and periodic nature of core media. Due to this reason, the results for E11 and E22 are similar to each other but also lower than depicted by FE results. Moreover values of G13 and G23 depicted by Ashby and Gibson are also simililar due to same reasons as mentioned above. The values of passion ratios are in close resemblance depicted by two techniques.

Chamis *et al.* performed finite element simulations on honeycomb cores by imposing displacement boundary conditions and readjusted the analytical results using solid mechanics theory to match the results with finite element results. Hence authors proposed simple equations matching results of finite element results. In his work, chamis *et al.* performed FE simulations on continuous honeycomb core model having number of hexagonal cells in length and width direction, whereas in the present work just one honeycomb core cell is modeled and results are obtained by applying periodic displacement boundary conditions. The out of plane modulus E33, two shear moduli G13 and G23 predicted by finite element method adopted in this study are in good agreement with the values obtained by Chamis *et al.*

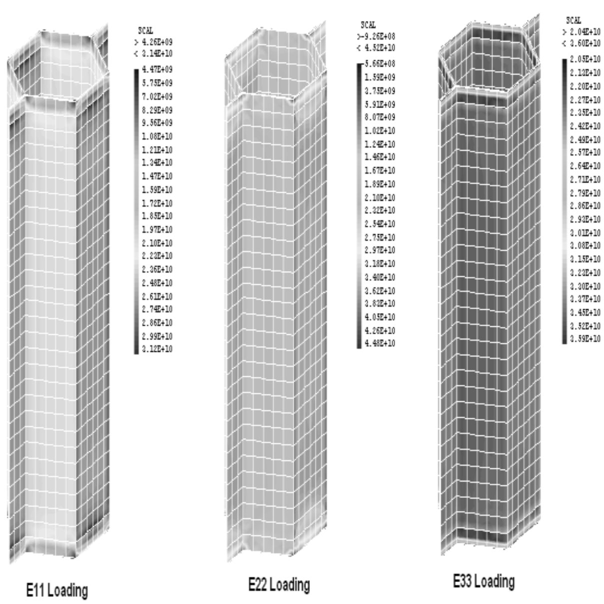

Fig. 4: Finite Element Analysis results of honeycomb core for three different loading conditions

Table 1: Comparison of results for honeycomb core Finite element analysis

	FEA Results	**Gibson and Ashby [2]**	**Chamis *et al.* [13]**
$E_{11}\left(\mathrm{N/m^2}\right)$	6.67E7	8.3E5	1E9
$E_{22}\left(\mathrm{N/m^2}\right)$	7.21E7	8.3E5	6.9E8
$E_{33}\left(\mathrm{N/m^2}\right)$	1.83E9	1.4E9	2.1E9
$G_{12}\left(\mathrm{N/m^2}\right)$	1.58E7	2E5	1.9E8
$G_{13}\left(\mathrm{N/m^2}\right)$	2.64E8	2.6E8	2.6E8
$G_{23}\left(\mathrm{N/m^2}\right)$	4.03E8	2.6E8	3.9E8
υ_{12}	0.886	1	0.33
υ_{13}	0.012	0	0.33
υ_{23}	0.013	0	0.33

Table 1: Comparison of results for honeycomb core Finite element analysis for the sandwich structure containing both honeycomb core and face sheet materials **(see Fig. 3)** is done for 21 different loading conditions to determine the 21 orthotropic coefficients. Finite element results of core material for three tensile loading conditions are shown in **Fig. 5.**

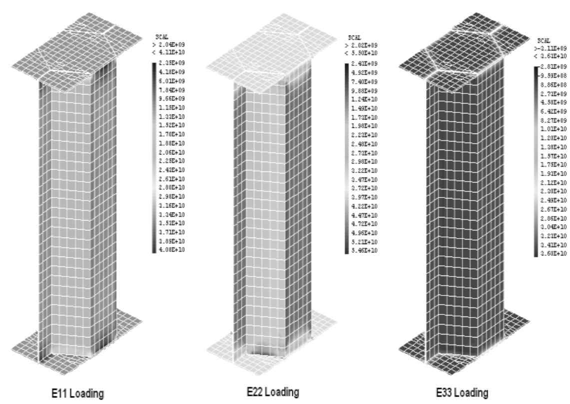

Fig. 5: Finite Element Analysis results of sandwich structure for three different loading conditions

The determined orthotropic properties of sandwich structure through homogenization process will be compared with properties determined by engineering empirical method. Here the engineering empirical method proposed by Wo is used to compute the equivalent elastic moduli of honeycomb sandwich structure. In the following equations (12)-(17), E_{f11}, E_{f22}, E_{f33}, G_{f12}, G_{f13} and G_{f23} are the elastic moduli of the lower and upper face sheets; E_{C11}, E_{C22}, E_{C33}, G_{C12}, G_{C13} and G_{C23} are the elastic moduli of honeycomb cores. If h_C, t_f and h are height of core, face sheet thickness and height of sandwich structure then:

$$E_{11} = E_{f11}\frac{2t_f}{h} + E_{C11}\frac{h_C}{h} \quad (12)$$

$$E_{22} = E_{f22}\frac{2t_f}{h} + E_{C22}\frac{h_C}{h} \quad (13)$$

$$\frac{1}{E_{33}} = \frac{2t_f}{E_{f33}h} + \frac{h_C}{E_{C33}h} \quad (14)$$

$$G_{12} = G_{f12}\frac{2t_f}{h} + G_{C12}\frac{h_C}{h} \quad (15)$$

$$\frac{1}{G_{13}} = \frac{2t_f}{G_{f13}h} + \frac{h_C}{G_{C13}h} \quad (16)$$

$$\frac{1}{G_{23}} = \frac{2t_f}{G_{f23}h} + \frac{h_C}{G_{C23}h} \quad (17)$$

Comparison of the results for the determined properties by Finite Element analysis and Wo for sandwich structure are given in **(Table 2).**

	FEA Results	**Wo [14]**
$E_{11}\left(\mathrm{N/m^2}\right)$	8.29E8	7.49E8
$E_{22}\left(\mathrm{N/m^2}\right)$	8.52E8	7.55E8
$E_{33}\left(\mathrm{N/m^2}\right)$	1.94E9	1.85E9
$G_{12}\left(\mathrm{N/m^2}\right)$	4.05E8	2.72E8
$G_{13}\left(\mathrm{N/m^2}\right)$	2.66E8	2.67E8
$G_{23}\left(\mathrm{N/m^2}\right)$	4.03E8	4.07E8

Table 2**:** Comparison of results for sandwich structure Finite element results for the sandwich structure based on proposed methodology in this paper are given in table 2 for three tensile moduli and for three shear moduli. The results obtained by analytical equations proposed by Wo are in good agreement with the finite element results.

5. CONCLUSION

In this article, strain energy based homogenization methodology is explained to determine the equivalent orthotropic properties of honeycomb sandwich structures. A 3D representative volume element (RVE) of one cell of honeycomb is selected to apply the homogenization technique rather selecting a continuous structure having multiple cells, hence considerably reducing the computational time. In the first part, homogenization is performed for honeycomb core material and determined orthotropic properties are compared with the properties based on available theories of different authors. In the second part, homogenization of sandwich structure containing core and face sheet is performed. The properties obtained by applying homogenization technique for sandwich structure and are successfully compared with analytical solution.

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