

GENERAL SOLUTIONS OF CONVECTIVE FLOWS OF
MHD CASSON FLUID WITH SLIP AND RADIATIVE
HEAT TRANSFER AT THE BOUNDARY

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SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF PHILOSOPHY
AT
UNIVERSITY OF MANAGEMENT AND TECHNOLOGY LAHORE
C-II JOHAR TOWN LAHORE
APRIL 2017

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UNIVERSITY OF MANAGEMENT AND TECHNOLOGY
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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled **“General Solutions Of Convective Flows of MHD Casson Fluid with Slip and Radiative Heat Transfer at the Boundary ”** by **Sana Ejaz** in partial fulfillment of the requirements for the degree of **Master of Philosophy**.

Dated: April 2017

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This dissertation is dedicated to my family.

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Abstract

The purpose of this work is to find General solutions for Magnetohydrodynamic(MHD) natural convection flow of an incompressible viscous fluid over an infinite vertical plate by considering Radiative heat transfer, Porous effects and slip conditions. These solutions, are obtained by Laplace transform technique. They satisfy all imposed initial and boundary conditions and generate large class of exact solutions. For illustration, three special cases are considered and some interesting results from literature are revived as limiting cases. The influence of different parameters on the fluid motion are graphically illustrated. This work has been published *International Journal of Computational Thermal Sciences*, **Vol. 9**, pp. 1-11, (2017).

Acknowledgements

By the grace of God Almighty, the Godhead and Lord of the universe, the most merciful and philanthropic, By the blessing of God Almighty who has given me this chance to be able to carry out this piece of work. All glorification and praise is for his beloved name.

I am highly grateful to my honored and commendable supervisor **Dr.M. Imran Asjad** for showing his enthusiastic guidance, sympathetic attitude, inexhaustable inspiration, and enlightened supervision. His efforts towards the inclusion of spirit of hard work and maintenance of professional integrity besides other valuable suggestions will always serve as a beacon of light throughout the course of my life.

No words to express my heartfelt and sincerest gratitude to my most honored teacher, **Dr. Muhammad Saeed** , Chairperson, Department of Mathematics, University of Management and Technology Lahore, who is gifted with all the traits of an excellent teacher and guide. His benevolent attention, keen and continued interest inestimable inspiration enabled me to attain my objectives without any difficulty. I wish to extend my gratefulness to my dignified teachers for there generous advices and moral support in the compilation of this dissertation.

I would love to dedicate my work to my parents especially my mother, who is an inspiration for me to continue my studies. My parents always helped and motivated me for higher studies.

Last but not the least I would like to thank my friends especially Rabia Nazeer and Shoaib Sardar for their support who always stood beside me in the difficult time. Their academic discussions and critical thinking have enlightened me.

SANA EJAZ

Chapter 1

Introduction

In physics, fluid dynamics is the branch of mechanics which describes flow of fluids(liquids and gases). It has various types, including hydrodynamics and aerodynamics.

Scientists beyond various branches have studied this type of mechanics. Fluid dynamics gives distinct approach for read ocean currents, weather setup, evolution of stars, tectonics and even flow of blood. A few salient industrial applications of fluid dynamics include wind farm, oil channels and air cooling systems. There are various applications of fluid mechanics. For example, flow of fluid in pipe and canals, form of all kinds of fluid machinery including fans, condensers , pumps, heat exchangers and rockets etc. Aerodynamics and hydrodynamics both are subdisciplines of fluid dynamics, have immense importance from the application point of view, to develop robotics. Latterly, for the review of linear and non linear problems, the engineers and scientist shows more interest in an analytical method .

Natural convection flows of Magnetohydrodynamic Casson fluid over an infinite vertical plate had studied widely due to its broad applications of ecological and engineering processes. It is also of great interest in industrial applications, various investigations