

THE DYNAMICAL STUDY OF COMPACT OBJECTS IN GENERAL RELATIVITY

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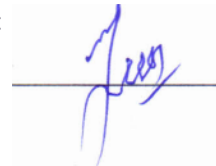


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Abstract

In this thesis, we discuss the dynamical stability of charged compact objects with the help of some mathematical models. For this purpose, we have selected three different models of charged compact objects to discuss the possible occurrence of cracking under different conditions. In first selected model, we discuss charged anisotropic compact objects with a linear equation of state. In second model, we study anisotropic charged compact object PSR J1614-2230 in quadratic regime, while in third model, we study charged compact stars corresponding to embedded class one metric with perfect inner fluid distribution. We investigate the impact of electromagnetic field on the stability regions of charged self-gravitating compact objects by using the concept of cracking. For this, we have applied local density perturbation scheme to the hydrostatic equilibrium equation as well as on physical parameters involved in the model. In particular, we have examined the cracking of charged compact objects (a) PSR J1614-2230, PSR J1903+327, Vela X-1, SMC X-1 and Cen X-3 with linear equation of state (b) PSR J1614-2230 with quadratic equation of state (c) Her X-1, PSR 1937+21, PSR J1614-2230, PSR J0348+0432 and RX J1856-37 corresponding to embedded class one metric. We conclude that these objects exhibit cracking and stability regions decreases with the increase of charge.

We also extend two conventional polytropic equations of state to generalized polytropic equations of state for spherical and cylindrical symmetries in the context of general relativity. For this purpose, we formulate the general framework to discuss the physical properties of spherical and cylindrical polytropes with charged anisotropic inner fluid distribution under conformally flat condition. We investigate the stability of generalized polytropic models through Tolman-mass and Whittaker formula for spherical and cylindrical symmetries respectively. We also discuss the possible occurrence of cracking in charged anisotropic polytropes developed under the assumption of generalized polytropic equation of state in two different ways (i) by carrying out local density perturbation under conformally flat condition (ii) by parametric perturbations. We conclude that one of the generalized polytropic equations of state results into a physically viable model and cracking appears for a specific range of density and model parameters.

Introduction

The ideology of general relativity (GR) is the relativistic gravity. The main theme of this theory lies in the structure of four dimensional spacetime, which describes various astrophysical phenomena like accelerating behavior of universe, black holes, pulsars, self-gravitation and gravitational collapse. Among all the astrophysical phenomena, the most important one is the gravitation collapse of astronomical objects. The self-gravitation can be define as gravitational force exerted on an object, or a system of objects, by the objects, which help them to bind together. The phenomenon of self-gravitation can be affected by various factor like change in atmospheric conditions, electromagnetism, gravitational force of nearby astronomical objects, which may lead towards gravitational collapse.