

**Third order Parallel Splitting Techniques**  
**For**  
**The Solution of Two Dimensional Heat Equation**  
**With Nonlocal Boundary Condition**



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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

*In the name of Allah  
the most Gracious  
the most Compassionate*

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Differential Equation . . . . .	1
1.2	Partial Differential Equation . . . . .	2
1.2.1	Quasi-Linear PDE's of 2nd Order . . . . .	2
1.2.2	Classification of Partial Differential Equations . . . . .	3
1.3	Finite Difference Methods . . . . .	4
1.3.1	Introduction . . . . .	4
1.3.2	First Order Forward Difference Formula . . . . .	5
1.3.3	First Order Backward Difference Formula . . . . .	6
1.3.4	Second-Order Central Difference Formula . . . . .	6
1.3.5	Some Common Finite Difference Schemes . . . . .	7
1.4	Stability . . . . .	8
1.5	Motivation and Objectives . . . . .	9
1.6	Applications . . . . .	10
<b>2</b>	<b>Literature Survey</b>	<b>12</b>
<b>3</b>	<b>Development of Third-Order Numerical Method For The Solution Of Heat Equations With Nonlocal Boundary Condition</b>	<b>15</b>
3.1	Discretization and Approximation for Space Derivatives . . . . .	16

3.1.1	A Rational Approximation For $exp(lB)$ . . . . .	23
3.1.2	Development of Algorithm . . . . .	27
3.2	Numerical Experiment . . . . .	28
3.3	Treatment of Nonlocal Boundary Condition . . . . .	29
3.3.1	Treatment of the nonlocal boundary condition by using Trapezoidal rule . . . . .	30
3.3.2	Treatment of the nonlocal boundary condition by using Simpson's 1/3 Rule . . . . .	31
3.3.3	Numerical Experiments . . . . .	34
3.4	Explanation of General Method For N=9 and Unknown $\eta(t)$ . . .	35
3.4.1	A Rational Approximation For $exp(lB)$ . . . . .	38
3.4.2	Treatment of The nonlocal boundary condition by using Simpson's 1/3 rule . . . . .	40
<b>4</b>	<b>Summary and Conclusion</b>	<b>59</b>

# List of Tables

3.1	Relative error when $\eta(t)$ is given . . . . .	49
3.2	Relative error when $\eta(t)$ is unknown . . . . .	50

# List of Figures

3.1	$N=49$ , $l=1/15000$ and $\eta$ is given . . . . .	51
3.2	$N=49$ , $l=1/15000$ and $\eta$ is un known . . . . .	52
3.3	$N=49$ $l=1/15000$ and $\eta$ is unknown . . . . .	53
3.4	$N=9$ , $l=1/10$ $\eta$ is unknown . . . . .	54
3.5	$N=9$ , $l=1/10$ $\eta$ is unknown . . . . .	55
3.6	$N=19$ , $l=1/20$ and $\eta$ is unknown . . . . .	56
3.7	$N=29$ , $l=1/30$ and $\eta$ is unknown . . . . .	57
3.8	$N=39$ , $l=1/40$ and $\eta$ is un known . . . . .	58

### **Abstract**

In this thesis our goal is to develop a third-order parallel splitting algorithm for solving linear partial differential equation in two dimensions with non local boundary condition. In this method third-order approximations are used to approximate spatial derivative and Simpson's 1/3 rule is used to approximate the non local boundary condition. Using this Parallel algorithm the results of numerical experiments are examined, presented and compared with the exact solution, as well as with the results already existed in the literature and found to be highly accurate.