

On Super (a, d) -Edge-Antimagic Total Labeling of A Class of Trees

A.Raheem, A.Q.Baig

Department of Mathematics, COMSATS Institute of Information Technology
Attock Campus, Pakistan

M.Javaid

Department of Mathematics, School of Science and Technology
University of Management and Technology (UMT), Lahore, Pakistan

E-mail: rahimciit7@gmail.com, aqbaig@gmail.com, muhammad.javaid@umt.edu.pk

Abstract: The concept of labeling has its origin in the works of Stewart (1966), Kotzig and Rosa (1970). Later on Enomoto, Llado, Nakamigawa and Ringel (1998) defined a super $(a, 0)$ -edge-antimagic total labeling and proposed the conjecture that every tree is a super $(a, 0)$ -edge-antimagic total graph. In the favour of this conjecture, the present paper deals with different results on antimagicness of a class of trees, which is called subdivided stars.

Key Words: Smarandachely super (a, d) -edge-antimagic total labeling, super (a, d) -edge-antimagic total labeling, stars and subdivision of stars.

AMS(2010): 53C78

§1. Introduction

All graphs in this paper are finite, undirected and simple. For a graph G , $V(G)$ and $E(G)$ denote the vertex-set and the edge-set, respectively. A (v, e) -graph G is a graph such that $|V(G)| = v$ and $|E(G)| = e$. A general reference for graph-theoretic ideas can be seen in [28]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper, the domain will be the set of all vertices and edges and such a labeling is called a *total labeling*. Some labelings use the vertex-set only or the edge-set only and we shall call them *vertex-labelings* or *edge-labelings*, respectively.

Definition 1.1 An (s, d) -edge-antimagic vertex (abbreviated to (s, d) -EAV) labeling of a (v, e) -graph G is a bijective function $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ such that the set of edge-sums of all edges in G , $\{w(xy) = \lambda(x) + \lambda(y) : xy \in E(G)\}$, forms an arithmetic progression $\{s, s + d, s + 2d, \dots, s + (e - 1)d\}$, where $s > 0$ and $d \geq 0$ are two fixed integers.

Furthermore, let $H \leq G$. If there is a bijective function $\lambda : V(H) \rightarrow \{1, 2, \dots, |H|\}$

¹Received October 23, 2013, Accepted December 2, 2014.

such that the set of edge-sums of all edges in H forms an arithmetic progression $\{s, s + d, s + 2d, \dots, s + (|E(H)| - 1)d\}$ but for all edges not in H is a constant, such a labeling is called a Smarandachely (s, d) -edge-antimagic labeling of G respect to H . Clearly, an (s, d) -EAV labeling of G is a Smarandachely (s, d) -EAV labeling of G respect to G itself.

Definition 1.2 A bijection $\lambda : V(G) \cup E(G) \rightarrow \{1, 2, \dots, v + e\}$ is called an (a, d) -edge-antimagic total $((a, d)$ -EAT) labeling of a (v, e) -graph G if the set of edge-weights $\{\lambda(x) + \lambda(xy) + \lambda(y) : xy \in E(G)\}$ forms an arithmetic progression starting from a and having common difference d , where $a > 0$ and $d \geq 0$ are two chosen integers. A graph that admits an (a, d) -EAT labeling is called an (a, d) -EAT graph.

Definition 1.3 If λ is an (a, d) -EAT labeling such that $\lambda(V(G)) = \{1, 2, \dots, v\}$ then λ is called a super (a, d) -EAT labeling and G is known as a super (a, d) -EAT graph.

In Definitions 1.2 and 1.3, if $d = 0$ then an $(a, 0)$ -EAT labeling is called an edge-magic total (EMT) labeling and a super $(a, 0)$ -EAT labeling is called a super edge magic total (SEMT) labeling. Moreover, in general a is called minimum edge-weight but particularly magic constant when $d = 0$. The definition of an (a, d) -EAT labeling was introduced by Simanjuntak, Bertault and Miller in [23] as a natural extension of *magic valuation* defined by Kotzig and Rosa [17-18]. A super (a, d) -EAT labeling is a natural extension of the notion of *super edge-magic labeling* defined by Enomoto, Llado, Nakamigawa and Ringel. Moreover, Enomoto et al. [8] proposed the following conjecture.

Conjecture 1.1 Every tree admits a super $(a, 0)$ -EAT labeling.

In the favor of this conjecture, many authors have considered a super $(a, 0)$ -EAT labeling for different particular classes of trees. Lee and Shah [19] verified this conjecture by a computer search for trees with at most 17 vertices. For different values of d , the results related to a super (a, d) -EAT labeling can be found for w-trees [13], stars [20], subdivided stars [14, 15, 21, 22, 29, 30], path-like trees [3], caterpillars [17, 18, 25], disjoint union of stars and books [10] and wheels, fans and friendship graphs [24], paths and cycles [23] and complete bipartite graphs [1]. For detail studies of a super (a, d) -EAT labeling reader can see [2, 4, 5, 7, 9-12].

Definition 1.4 Let $n_i \geq 1$, $1 \leq i \leq r$, and $r \geq 2$. A subdivided star $T(n_1, n_2, \dots, n_r)$ is a tree obtained by inserting $n_i - 1$ vertices to each of the i th edge of the star $K_{1,r}$. Moreover suppose that $V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r; 1 \leq l_i \leq n_i\}$ is the vertex-set and $E(G) = \{cx_i^1 | 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} | 1 \leq i \leq r; 1 \leq l_i \leq n_i - 1\}$ is the edge-set of the subdivided star $G \cong T(n_1, n_2, \dots, n_r)$ then $v = \sum_{i=1}^r n_i + 1$ and $e = \sum_{i=1}^r n_i$.

Lu [29,30] called the subdivided star $T(n_1, n_2, n_3)$ as a three-path tree and proved that it is a super $(a, 0)$ -EAT graph if n_1 and n_2 are odd with $n_3 = n_2 + 1$ or $n_3 = n_2 + 2$. Ngurah et al. [21] proved that the subdivided star $T(n_1, n_2, n_3)$ is also a super $(a, 0)$ -EAT graph if $n_3 = n_2 + 3$ or $n_3 = n_2 + 4$. Salman et al. [22] found a super $(a, 0)$ -EAT labeling on the subdivided stars $T(\underbrace{n, n, n, \dots, n}_{r\text{-times}})$, where $n \in \{2, 3\}$.

Moreover, Javaid et al. [14,15] proved the following results related to a super (a, d) -EAT labeling on different subclasses of subdivided stars for different values of d :

- For any odd $n \geq 3$, $G \cong T(n, n-1, n, n)$ admits a super $(a, 0)$ -EAT labeling with $a = 10n + 2$;
- For any odd $n \geq 3$ and $m \geq 3$, $G \cong T(n, n, m, m)$ admits a super $(a, 0)$ -EAT labeling with $a = 6n + 5m + 2$;
- For any odd $n \geq 3$ and $p \geq 5$, $G \cong T(n, n, n+2, n+2, n_5, \dots, n_p)$ admits a super $(a, 0)$ -EAT labeling with $a = 2v + s - 1$, a super $(a, 1)$ -EAT labeling with $a = s + \frac{3}{2}v$ and a super $(a, 2)$ -EAT labeling with $a = v + s + 1$ where $v = |V(G)|$, $s = (2n + 6) + \sum_{m=5}^p [(n+1)2^{m-5} + 1]$ and $n_r = 1 + (n+1)2^{r-4}$ for $5 \leq r \leq p$.

However, the investigation of the different results related to a super (a, d) -EAT labeling of the subdivided star $T(n_1, n_2, n_3, \dots, n_r)$ for $n_1 \neq n_2 \neq n_3, \dots, \neq n_r$ is still open. In this paper, for $d \in \{0, 1, 2\}$, we formulate a super (a, d) -EAT labeling on the subclasses of subdivided stars denoted by $T(kn, kn, kn, kn, 2kn, n_6, \dots, n_r)$ and $T(kn, kn, 2n, 2n+2, n_5, \dots, n_r)$ under certain conditions.

§2. Basic Results

In this section, we present some basic results which will be used frequently in the main results. Ngurah et al. [21] found lower and upper bounds of the minimum edge-weight a for a subclass of the subdivided stars, which is stated as follows:

Lemma 2.1 *If $T(n_1, n_2, n_3)$ is a super $(a, 0)$ -EAT graph, then $\frac{1}{2l}(5l^2 + 3l + 6) \leq a \leq \frac{1}{2l}(5l^2 + 11l - 6)$, where $l = \sum_{i=1}^3 n_i$.*

The lower and upper bounds of the minimum edge-weight a for another subclass of subdivided stars established by Salman et al. [22] are given below:

Lemma 2.2 *If $T(\underbrace{n, n, \dots, n}_{n\text{-times}})$ is a super $(a, 0)$ -EAT graph, then $\frac{1}{2l}(5l^2 + (9 - 2n)l + n^2 - n) \leq a \leq \frac{1}{2l}(5l^2 + (2n + 5)l + n - n^2)$, where $l = n^2$.*

Moreover, the following lemma presents the lower and upper bound of the minimum edge-weight a for the most generalized subclass of subdivided stars proved by Javaid and Akhlaq:

Lemma 2.3([16]) *If $T(n_1, n_2, n_3, \dots, n_r)$ has a super (a, d) -EAT labeling, then $\frac{1}{2l}(5l^2 + r^2 - 2lr + 9l - r - (l-1)ld) \leq a \leq \frac{1}{2l}(5l^2 - r^2 + 2lr + 5l + r - (l-1)ld)$, where $l = \sum_{i=1}^r n_i$ and $d \in \{0, 1, 2, 3\}$.*

Bača and Miller [4] state a necessary condition for a graph to be super (a, d) -EAT, which

provides an upper bound on the parameter d . Let a (v, e) -graph G be a super (a, d) -EAT. The minimum possible edge-weight is at least $v + 4$. The maximum possible edge-weight is no more than $3v + e - 1$. Thus $a + (e - 1)d \leq 3v + e - 1$ or $d \leq \frac{2v + e - 5}{e - 1}$. For any subdivided star, where $v = e + 1$, it follows that $d \leq 3$.

Let us consider the following proposition which we will use frequently in the main results.

Proposition 2.1([3]) *If a (v, e) -graph G has a (s, d) -EAV labeling then*

- (1) G has a super $(s + v + 1, d + 1)$ -EAT labeling;
- (2) G has a super $(s + v + e, d - 1)$ -EAT labeling.

§3. Super (a, d) -EAT Labeling of Subdivided Stars

Theorem 3.1 *For any even $n \geq 2$ and $r \geq 6$, $G \cong T(n + 3, n + 2, n, n + 1, 2n + 1, n_6, \dots, n_r)$ admits a super $(a, 0)$ -edge-antimagic total labeling with $a = 2v + s - 1$ and a super $(a, 2)$ -edge-antimagic total labeling with $a = v + s + 1$ where $v = |V(G)|$, $s = (3n + 7) + \sum_{m=6}^r [2^{m-5}n + 1]$ and $n_m = 2^{m-4}n + 1$ for $6 \leq m \leq r$.*

Proof Let us denote the vertices and edges of G , as follows:

$$V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r; 1 \leq l_i \leq n_i\},$$

$$E(G) = \{cx_i^1 | 1 \leq i \leq r\} \cup \{x_i^{l_i}x_i^{l_i+1} | 1 \leq i \leq r; 1 \leq l_i \leq n_i - 1\}.$$

If $v = |V(G)|$ and $e = |E(G)|$, then

$$v = (6n + 8) + \sum_{m=6}^r [2^{m-6}4n + 1] \quad \text{and} \quad e = v - 1.$$

Now, we define the labeling $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as follows:

$$\lambda(c) = (4n + 8) + \sum_{m=6}^r [2^{m-6}2n + 1].$$

For odd $1 \leq l_i \leq n_i$, where $i = 1, 2, 3, 4, 5$ and $6 \leq i \leq r$, we define

$$\lambda(u) = \begin{cases} \frac{l_1 + 1}{2}, & \text{for } u = x_1^{l_1}, \\ n + 3 - \frac{l_2 - 1}{2}, & \text{for } u = x_2^{l_2}, \\ (n + 4) + \frac{l_3 - 1}{2}, & \text{for } u = x_3^{l_3}, \\ (2n + 4) - \frac{l_4 - 1}{2}, & \text{for } u = x_4^{l_4}, \\ (3n + 5) - \frac{l_5 - 1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (3n + 5) + \sum_{m=6}^i [2^{m-6}2n + 1] - \frac{l_i - 1}{2},$$

respectively. For even $1 \leq l_i \leq n_i$, $\alpha = (3n + 5) + \sum_{m=6}^r [2^{m-6}2n + 1]$, $i = 1, 2, 3, 4, 5$ and $6 \leq i \leq r$, we define

$$\lambda(u) = \begin{cases} (\alpha + 1) + \frac{l_1 - 2}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha + n + 2) - \frac{l_2 - 2}{2}, & \text{for } u = x_2^{l_2}, \\ (\alpha + n + 4) + \frac{l_3 - 2}{2}, & \text{for } u = x_3^{l_3}, \\ (\alpha + 2n + 3) - \frac{l_4 - 2}{2}, & \text{for } u = x_4^{l_4}, \\ (\alpha + 3n + 3) - \frac{l_5 - 2}{2}, & \text{for } u = x_5^{l_5} \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (\alpha + 3n + 3) + \sum_{m=6}^i [2^{m-6}2n] - \frac{l_i - 2}{2},$$

respectively.

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = \alpha + 2, \alpha + 3, \dots, \alpha + 1 + e$. Therefore, by Lemma 2.1, λ can be extended to a super $(a, 0)$ -edge-antimagic total labeling and we obtain the magic constant $a = v + e + s = 2v + (3n + 6) + \sum_{m=6}^r [2^{m-6}2n + 1]$.

Similarly by Lemma 2.2, λ can be extended to a super $(a, 2)$ -edge-antimagic total labeling and we obtain the magic constant $a = v + 1 + s = v + (3n + 8) + \sum_{m=6}^r [2^{m-6}2n + 1]$. \square

Theorem 3.2 For any odd $n \geq 3$ and $r \geq 6$, $G \cong T(n + 3, n + 2, n, n + 1, 2n + 1, n_6, \dots, n_r)$ admits a super $(a, 1)$ -edge-antimagic total labeling with $a = s + \frac{3v}{2}$ if v is even, where $v = |V(G)|$, $s = (3n + 7) + \sum_{m=6}^r [2^{m-5}n + 1]$ and $n_m = 2^{m-4}n + 1$ for $6 \leq m \leq r$.

Proof Let us consider the vertices and edges of G , as defined in Theorem 2.6. Now, we define the labeling $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as in same theorem. It follows that the edge-weights of all edges of G constitute an arithmetic sequence $s = \alpha + 2, \alpha + 3, \dots, \alpha + 1 + e$ with common difference 1, where

$$\alpha = (3n + 5) + \sum_{m=6}^r [2^{m-6}2n + 1].$$

We denote it by $A = \{a_i; 1 \leq i \leq e\}$. Now for G we complete the edge labeling λ for super $(a, 1)$ -edge-antimagic total labeling with values in the arithmetic sequence $v + 1, v + 2, \dots, v + e$ with common difference 1. Let us denote it by $B = \{b_j; 1 \leq j \leq e\}$. Define $C = \{a_{2i-1} + b_{e-i+1}; 1 \leq i \leq \frac{e+1}{2}\} \cup \{a_{2j} + b_{\frac{e-1}{2}-j+1}; 1 \leq j \leq \frac{e+1}{2} - 1\}$. It is easy to see

that C constitutes an arithmetic sequence with $d = 1$ and

$$a = s + \frac{3v}{2} = (12n + 19) + \frac{1}{2} \sum_{m=6}^r [2^{m-3}2n + 5].$$

Since all vertices receive the smallest labels, λ is a super $(a, 1)$ -edge-antimagic total labeling. \square

Theorem 3.3 For any even $n \geq 2$ and $r \geq 6$, $G \cong T(n + 2, n, n, n + 1, 2(n + 1), n_6, \dots, n_r)$ admits a super $(a, 0)$ -edge-antimagic total labeling with $a = 2v + s - 1$ and a super $(a, 2)$ -edge-antimagic total labeling with $a = v + s + 1$ where $v = |V(G)|$, $s = (3n + 5) + \sum_{m=6}^r [2^{m-5}n + 2]$ and $n_m = 2^{m-4}n + 2$ for $6 \leq m \leq r$.

Proof Let us denote the vertices and edges of G as follows:

$$V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r; 1 \leq l_i \leq n_i\};$$

$$E(G) = \{cx_i^1 | 1 \leq i \leq r\} \cup \{x_i^{l_i}x_i^{l_i+1} | 1 \leq i \leq r; 1 \leq l_i \leq n_i - 1\}.$$

If $v = |V(G)|$ and $e = |E(G)|$, then

$$v = (6n + 6) + \sum_{m=6}^r [2^{m-6}4(n+)] \quad \text{and} \quad e = v - 1.$$

Now, we define the labeling $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as follows:

$$\lambda(c) = (4n + 5) + \sum_{m=6}^r [2^{m-6}2n + 2].$$

For odd $1 \leq l_i \leq n_i$, where $i = 1, 2, 3, 4, 5$ and $6 \leq i \leq r$, we define

$$\lambda(u) = \begin{cases} \frac{l_1 + 1}{2}, & \text{for } u = x_1^{l_1}, \\ n + 1 - \frac{l_2 - 1}{2}, & \text{for } u = x_2^{l_2}, \\ (n + 2) - \frac{l_3 + 1}{2}, & \text{for } u = x_3^{l_3}, \\ (2n + 2) - \frac{l_4 - 1}{2}, & \text{for } u = x_4^{l_4}, \\ (3n + 3) - \frac{l_5 - 1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

$$\lambda(x_i^{l_i}) = (3n + 3) + \sum_{m=6}^i [2^{m-6}2n + 2] - \frac{l_i - 1}{2},$$

respectively. For even $1 \leq l_i \leq n_i$, $\alpha = (3n + 43) + \sum_{m=6}^r [2^{m-6}2n + 2]$, $i = 1, 2, 3, 4, 5$ and

$5 \leq i \leq r$, we define

$$\lambda(u) = \begin{cases} (\alpha + 1) + \frac{l_1 - 2}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha + n(\alpha + n + 1) - \frac{l_2 - 2}{2}), & \text{for } u = x_2^{l_2}, \\ (\alpha + n + 3) - \frac{l_3 - 2}{2}, & \text{for } u = x_3^{l_3}, \\ (\alpha + 2n + 2) - \frac{l_4 - 2}{2}, & \text{for } u = x_4^{l_4}, \\ (\alpha + 3n + 3) - \frac{l_5 - 2}{2}, & \text{for } u = x_5^{l_5} \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (\alpha + 3n + 3) + \sum_{m=6}^i [2^{m-6}4(n+1)] - \frac{l_i - 2}{2},$$

respectively.

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = \alpha + 2, \alpha + 3, \dots, \alpha + 1 + e$. Therefore, by Lemma 2.1, λ can be extended to a super $(a, 0)$ -edge-antimagic total labeling and we obtain the magic constant

$$a = v + e + s = 2v + (3n + 4) + \sum_{m=6}^r [2^{m-6}2n + 2].$$

Similarly by Lemma 2.2, λ can be extended to a super $(a, 2)$ -edge-antimagic total labeling and we obtain the magic constant $a = v + 1 + s = v + (3n + 6) + \sum_{m=6}^r [2^{m-6}2n + 2]$. \square

Theorem 3.4 For any odd $n \geq 3$ and $r \geq 6$, $G \cong T(n + 2, n, n, n + 1, 2(n + 1), n_6, \dots, n_r)$ admits a super $(a, 1)$ -edge-antimagic total labeling with $a = s + \frac{3v}{2}$ if v is even, where $v = |V(G)|$, $s = (3n + 5) + \sum_{m=6}^r [2^{m-5}n + 2]$ and $n_m = 2^{m-4}n + 2$ for $6 \leq m \leq r$.

Proof Let us consider the vertices and edges of G , as defined in Theorem 2.6. Now, we define the labeling $\lambda : V(G) \rightarrow \{1, 2, \dots, v\}$ as in same theorem. It follows that the edge-weights of all edges of G constitute an arithmetic sequence $s = \alpha + 2, \alpha + 3, \dots, \alpha + 1 + e$ with common difference 1, where

$$\alpha = (3n + 3) + \sum_{m=6}^r [2^{m-6}2(n + 1)].$$

We denote it by $A = \{a_i; 1 \leq i \leq e\}$. Now for G we complete the edge labeling λ for super $(a, 1)$ -edge-antimagic total labeling with values in the arithmetic sequence $v + 1, v + 2, \dots, v + e$ with common difference 1. Let us denote it by $B = \{b_j; 1 \leq j \leq e\}$. Define $C = \{a_{2i-1} + b_{e-i+1}; 1 \leq i \leq \frac{e+1}{2}\} \cup \{a_{2j} + b_{\frac{e-1}{2}-j+1}; 1 \leq j \leq \frac{e+1}{2} - 1\}$. It is easy to see

that C constitutes an arithmetic sequence with $d = 1$ and

$$a = s + \frac{3v}{2} = (12n + 14) + \sum_{m=6}^r [2^{m-5}(4n + 3) + 2].$$

Since all vertices receive the smallest labels, λ is a super $(a, 1)$ -edge-antimagic total labeling. \square

§4. Conclusion

In this paper, we have shown that two different subclasses of subdivided stars admit a super (a, d) -EAT labeling for $d \in \{0, 1, 2\}$. However, the problem is still open for the magicness of $T(n_1, n_2, n_3, \dots, n_r)$, where $n_i = n$ and $1 \leq i \leq r$.

References

- [1] Bača M., Y.Lin, M.Miller and M.Z.Youssef, Edge-antimagic graphs, *Discrete Math.*, 307(2007), 1232–1244.
- [2] Bača M., Y.Lin, M.Miller and R.Simanjuntak, New constructions of magic and antimagic graph labelings, *Utilitas Math.*, 60(2001), 229–239.
- [3] Bača M., Y.Lin and F.A.Muntaner-Batle, Super edge-antimagic labelings of the path-like trees, *Utilitas Math.*, 73(2007), 117–128.
- [4] Bača M. and M. Miller, *Super Edge-Antimagic Graphs*, Brown Walker Press, Boca Raton, Florida USA, 2008.
- [5] Bača M., A.Semaničová -Feňovčíková and M.K.Shafiq, A method to generate large classes of edge-antimagic trees, *Utilitas Math.*, 86(2011), 33–43.
- [6] Baskoro E.T., I.W.Sudarsana and Y.M.Cholily, How to construct new super edge-magic graphs from some old ones, *J. Indones. Math. Soc. (MIHIM)*, **11:2** (2005), 155–162.
- [7] Dafik, M.Miller, J.Ryan and M.Bača, On super (a, d) -edge antimagic total labeling of disconnected graphs, *Discrete Math.*, 309 (2009), 4909–4915.
- [8] Enomoto H., A.S.Lladó, T.Nakamigawa and G.Ringel, Super edge-magic graphs, *SUT J. Math.*, 34 (1998), 105–109.
- [9] Figueroa-Centeno R.M., R.Ichishima and F.A.Muntaner-Batle, The place of super edge-magic labelings among other classes of labelings, *Discrete Math.*, 231 (2001), 153–168.
- [10] Figueroa-Centeno R.M., R.Ichishima and F.A. Muntaner-Batle, On super edge-magic graph, *Ars Combinatoria*, 64 (2002), 81–95.
- [11] Fukuchi Y., A recursive theorem for super edge-magic labeling of trees, *SUT J. Math.*, 36 (2000), 279–285.
- [12] Gallian J.A., A dynamic survey of graph labeling, *Electron. J. Combin.*, **17** (2010).
- [13] Javaid M., M.Hussain, K.Ali and K.H.Dar, Super edge-magic total labeling on w-trees, *Utilitas Math.*, 86 (2011), 183–191.
- [14] Javaid M., M.Hussain, K.Ali and H.Shaker, On super edge-magic total labeling on subdivision of trees, *Utilitas Math.*, 89 (2012), 169–177.

- [15] Javaid M. and A.A.Bhatti, On super (a, d) -edge-antimagic total labeling of subdivided stars, *Utilitas Math.*, 105 (2012), 503–512.
- [16] Javaid M. and A.A.Bhatti, Super (a, d) -edge-antimagic total labeling of subdivided stars and w-trees, *Utilitas Math.*, to appear.
- [17] Kotzig A. and A.Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.*, 13 (1970), 451–461.
- [18] Kotzig A. and A.Rosa, Magic valuation of complete graphs, *Centre de Recherches Mathematiques, Universite de Montreal*, (1972), CRM-175.
- [19] Lee S.M. and Q.X.Shah, All trees with at most 17 vertices are super edge-magic, *16th MCCCC Conference, Carbondale*, University Southern Illinois, November (2002).
- [20] Lee S.M. and M.C.Kong, On super edge-magic n stars, *J. Combin. Math. Combin. Comput.*, 42 (2002), 81–96.
- [21] Ngurah A.A.G., R.Simanjuntak and E.T.Baskoro, On (super) edge-magic total labeling of subdivision of $K_{1,3}$, *SUT J. Math.*, 43 (2007), 127–136.
- [22] Salman A.N.M., A.A.G.Ngurah and N.Izzati, On super edge-magic total labeling of a subdivision of a star S_n , *Utilitas Mathematica*, 81 (2010), 275–284.
- [23] Simanjuntak R., F.Bertault and M.Miller, Two new (a, d) -antimagic graph labelings, *Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms*, 11 (2000), 179–189.
- [24] Slamin, M. Bača, Y.Lin, M.Miller and R.Simanjuntak, Edge-magic total labelings of wheel, fans and friendship graphs, *Bull. ICA*, 35 (2002), 89–98.
- [25] Sugeng K.A., M.Miller, Slamin and M.Bača, (a, d) -edge-antimagic total labelings of caterpillars, *Lecture Notes Comput. Sci.*, 3330 (2005), 169–180.
- [26] Stewart M.B., Supermagic complete graphs, *Can. J. Math.*, 19 (1966): 427-438.
- [27] Wallis W.D., *Magic Graphs*, Birkhauser, Boston-Basel-Berlin, 2001.
- [28] West D. B., *An Introduction to Graph Theory*, Prentice-Hall, 1996.
- [29] Yong-Ji Lu, A proof of three-path trees $P(m, n, t)$ being edge-magic, *College Mathematica*, **17:2** (2001), 41–44.
- [30] Yong-Ji Lu, A proof of three-path trees $P(m, n, t)$ being edge-magic (II), *College Mathematica*, **20:3** (2004), 51–53.