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Lemaitre–Tolman–Bondi dust cloud collapse in Brans–Dicke gravity

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This paper investigates the phenomenon of gravitational collapse of Lemaitre–Tolman–Bondi (LTB) model in the presence of Brans–Dicke (BD) scalar field with nonzero potential field. We find a class of solutions by taking perfect fluid as well as scalar field and check the validity of weak energy conditions. It turns out that two different types of singularities are formed in the presence of scalar field. We conclude that the end state of gravitational collapse turns out to be a black hole (BH) contrary to general relativity (GR).

Keywords: Brans–Dicke theory; scalar field; gravitational collapse.

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1. Introduction

When a massive star (twice the mass of the Sun) consumes its nuclear fuel, it collapses under its own gravity. According to singularity theorems,[1] the collapse of a star leads to the formation of spacetime singularity. The nature of this singularity has been remained an important issue in GR. Penrose[2] presented a hypothesis known as cosmic censorship hypothesis (CCH), which states that singularity evolving from regular initial conditions must be hidden within event horizon, i.e. no naked singularity exists. Many people[3–12] investigated the formation of naked singularity (violation of CCH) in the last few decades. It is found that the nature of singularity depends on the time formation of apparent horizon. If the singularity is formed before the formation of apparent horizon, it becomes visible to outside observer, i.e. a naked singularity. If apparent horizon is formed earlier than the formation of singularity, then the singularity must be covered within the event horizon.

In this context, LTB model[13–16] is extensively used to describe the final fate of gravitational collapse in GR. The LTB model is an inhomogeneous spherical symmetric dust model which leads to two different types of singularities: shell crossing and shell focusing. The shell focusing singularities can further be divided into central shell focusing singularity (visible) and non-central shell focusing singularity (hidden).

In weak-field regime, all theories of gravity (including GR) are consistent with weak-field gravitational test, but in strong-field regime, modified theories may deviate widely from GR. It is believed that experiments and observations in strong-field regime can lead to the development of suitable modified theory of gravity. The phenomenon of gravitational collapse is one such example of strong-field regime. Thus the study of collapse phenomenon in any modified gravity may enhance and strengthen the structure of that theory. Moreover, the dynamics of modified theories may modify the dynamics of gravitational collapse.[17–22]

In this context, the scalar–tensor theories have emerged with dominant features in strong-field regime.[18–26] The BD theory[27–32] is the most explored example of this theory which has performed a leading role in its development. The BD gravity has been introduced by assuming dynamical gravitational constant $\left(G = \frac{G_0}{\phi}\right)$, where ϕ is a massless scalar field which varies according to time and position. This theory also involves a constant tuneable parameter ω_{BD}, known as BD coupling constant. It is consistent with many observations and experiments of solar system as well as other gravitational systems for $|\omega_{\mathrm{BD}}| > 40{,}000$.[33–36] Moreover, the BD gravity involves more degrees of freedom than GR and hence it admits large number of solutions of the field equations. In particular, Birkhoff's theorem does not remain valid in BD theory[37] in a way variety of both static and non-static solutions are obtained in spherical symmetric vacuum situations.

Matsuda and Nariai[38] were the first who explored numerically the spherically symmetric collapse of an ideal gas in BD gravity. Shibata *et al.*[18] discussed the radiation of scalar gravitational waves from inhomogeneous dust collapse which are not found in GR. Scheel *et al.*[24,25] described Openheimer–Snyder collapse in BD scalar field which leads to the formation of BH for certain range of ω_{BD}. Harada *et al.*[26] found that Openheimer–Snyder collapse in BD gravity provides deviation from GR such as the violation of second law of BH thermodynamics. Ziaie *et al.*[39] demonstrated a class of Friedmann collapse solutions in BD gravity and showed that for different values of w, both types of singularities (hidden or naked) are obtained.

In this paper, we investigate the final fate of a class of solutions of spherical symmetric inhomogeneous LTB model collapse in BD gravity. The paper is organized in the following format. In Sec. 2, we discuss LTB model solutions with perfect fluid in BD gravity and check the validity of weak energy conditions. Section 3 is devoted to the formation of singularities and their respective properties. We also discuss the end state of collapse (BH or naked singularity), the behavior of scalar field as well as potential. The last section summarizes the results.

2. Collapse Solutions

In this section, we construct a class of dust collapse solutions in BD gravity. The action of self-interacting BD theory with scalar field ϕ is[27–32]

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega_{\rm BD}}{\phi}\nabla^\alpha\phi\nabla_\alpha\phi - U(\phi) + L_m\right), \tag{1}$$

where $\omega_{\rm BD}$, $U(\phi)$ and L_m represent constant BD coupling parameter, self-interacting potential and matter contribution, respectively. The respective BD field equations are described by

$$G_{\mu\nu} = T^{(\rm eff)}_{\mu\nu} = \frac{1}{\phi}(T^m_{\mu\nu} + T^\phi_{\mu\nu})\,. \tag{2}$$

Here $G_{\mu\nu}$ is the Einstein tensor, $T^{(\rm eff)}_{\mu\nu}$ is the effective stress energy tensor with $T^m_{\mu\nu}$ and $T^\phi_{\mu\nu}$ represent the stress energy tensors of matter fluid and scalar field, respectively given as follows:

$$T^{(\rm eff)}_{\mu\nu} = (\rho_{(\rm eff)} + P_{(\rm eff)})V_\mu V_\nu + P_{(\rm eff)}g_{\mu\nu}\,, \tag{3}$$

$$T^\phi_{\mu\nu} = [\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi] + \frac{\omega_{\rm BD}}{\phi}\left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right] - \frac{U(\phi)}{2}g_{\mu\nu}\,, \tag{4}$$

$$T^m_{\mu\nu} = \rho_m V_\mu V_\nu\,, \qquad \mu,\nu = 0,1,2,3\,, \tag{5}$$

where $\rho_{(\rm eff)}$, $P_{(\rm eff)}$ and ρ_m are referred to effective energy density, effective pressure and matter density, respectively, V_μ is the four velocity. The variation of action (1) with respect to scalar field gives

$$\Box\phi = \frac{T^m}{3+2\omega_{\rm BD}} + \frac{1}{3+2\omega_{\rm BD}}\left[\phi\frac{dU(\phi)}{d\phi} - 2U(\phi)\right], \tag{6}$$

where T^m is the trace of $T^m_{\mu\nu}$.

In order to discuss inhomogeneous dust collapse in BD gravity, we consider LTB spacetime[3–16] as an interior geometry of the collapsing cloud. The line element of LTB spacetime is given by

$$ds^2 = -dt^2 + e^{\lambda(t,r)}\,dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta\,d\varphi^2)\,. \tag{7}$$

Here t and r are co-moving coordinates, the coordinate r labels the spherical shells of cloud, $R(t,r)$ is the physical radius of such a mass shell. The proper area of a spherical shell is given by $4\pi R^2$ which tends to zero for $R(t,r) = 0$ when r is constant. Using Eqs. (2) and (7), we have

$$G^t_r = \frac{e^{-\lambda}}{R}(2\dot{R}' - \dot{\lambda}R') = T^{t(\rm eff)}_r = 0\,,$$

where prime and dot indicate partial derivative with respect to r and t. Integration of this equation yields

$$e^{\lambda(t,r)} = \frac{R'^2}{f(r)}\,, \tag{8}$$

where $f(r)$ is an integration function. Thus the line element (7) becomes

$$ds^2 = -dt^2 + \frac{R'^2}{f(r)}dr^2 + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,. \tag{9}$$

When the particles follow non-intersecting geodesics due to the absence of matter pressure, one can divide the spacetime into space and time in a very significant way such that at the surface $t = \text{const.}$, the 3-space model,

$$ds^2 = \frac{R'^2}{f(r)}dr^2 + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,,$$

is flat if and only if $f(r) = 1$.[16] Since in BD gravity, the scalar field is not allowed to interact directly with matter, so pressure due to scalar field does not disturb the geodesics of particles (they remain non-intersecting). Therefore, the above defined 3-space model remains flat for $f(r) = 1$ in BD gravity. Consequently, the line element (9) turns out to be

$$ds^2 = -dt^2 + R'^2(t,r)dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{10}$$

which is a flat model of LTB spacetime in BD gravity. The corresponding BD field equations (2) are given as follows:

$$G_t^t = T_t^{t(\text{eff})} = \rho_{(\text{eff})} = \frac{1}{\phi}(T_t^{t(m)} + T_t^{t(\phi)}) = \frac{1}{\phi}(\rho_m + \rho_\phi) = \frac{\rho_m}{\phi} + \frac{\omega_{\text{BD}}}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\omega_{\text{BD}}}{2}\frac{\phi'^2}{\phi^2} - \frac{(R'R^2)^{\cdot}\dot{\phi}}{R'R^2\phi} + \left(\frac{R^2}{R'}\right)'\frac{\phi'}{R'R^2\phi} + \frac{\phi''}{\phi R'} + \frac{U(\phi)}{2}\,, \tag{11}$$

$$G_r^r = T_r^{r(\text{eff})} = P_r^{(\text{eff})} = \frac{1}{\phi}(T_r^{r(m)} + T_r^{r(\phi)}) = \frac{P_r^\phi}{\phi} = \frac{\omega_{\text{BD}}}{2}\frac{R^2\dot{\phi}^2}{\phi^2} - \frac{\omega_{\text{BD}}}{2}\frac{\phi'^2}{\phi^2} - \frac{R^2\ddot{\phi}}{\phi} + \frac{\phi'^2}{\phi} - \frac{(R'R^2)^{\cdot}\dot{\phi}}{R^2\phi} + \frac{\phi''}{\phi} + \left(\frac{R^2}{R'}\right)'\frac{R'\phi'}{R^2\phi} - \frac{U(\phi)}{2}\,, \tag{12}$$

$$G_\theta^\theta = T_\theta^{\theta(\text{eff})} = P_\theta^{(\text{eff})} = G_\varphi^\varphi = T_\varphi^{\varphi(\text{eff})} = P_\varphi^{(\text{eff})} = \frac{1}{\phi}(T_\varphi^{\varphi(m)} + T_\varphi^{\varphi(\phi)}) = \frac{P_\varphi^\phi}{\phi} = \frac{\omega_{\text{BD}}}{2}\frac{R^2\dot{\phi}^2}{\phi^2} - \frac{\omega_{\text{BD}}}{2}\frac{R^2R'^2\phi'^2}{\phi^2} + \frac{R^2\ddot{\phi}}{\phi} + \frac{(R'R^2)^{\cdot}\dot{\phi}}{R'\phi} - \frac{R^2\phi''}{R'\phi} - \frac{\phi'}{R'\phi}\left(\frac{R^2}{R'}\right)' - \frac{U(\phi)}{2}\,, \tag{13}$$

$$G^t_r = T^{t(\mathrm{eff})}_r = 0 = \frac{1}{\phi} T^{t(\phi)}_r = \frac{1}{\phi}\left(\dot{\phi}' - \frac{\dot{R}'}{R'}\phi'\right) + \frac{\omega_{\mathrm{BD}}\dot{\phi}\phi'}{\phi^2}\,, \tag{14}$$

where ρ_ϕ, P^ϕ_r and P^ϕ_φ are the scalar field density, radial and transversal pressures, respectively. Since the scalar as well as matter field contents are perfect fluid, so $P^{(\mathrm{eff})}_r = P^{(\mathrm{eff})}_\theta = P^{(\mathrm{eff})}_\varphi = P^{(\mathrm{eff})}$ and $P_{(\mathrm{eff})} = K_{\mathrm{BD}}\rho_{(\mathrm{eff})}$, where $K_{\mathrm{BD}} = \frac{P_{(\mathrm{eff})}}{\rho_{(\mathrm{eff})}}$ is an equation-of-state parameter. The solutions of Eqs. (11)–(13) are[39,40]

$$\rho_{(\mathrm{eff})} = \frac{F'}{R^2R'}\,, \qquad P^{(\mathrm{eff})} = \frac{-\dot{F}}{R^2\dot{R}}\,, \qquad \dot{R}^2 = \frac{F}{R}\,. \tag{15}$$

Here $F(t,r)$ is the Misner–Sharp mass function[41] which can be interpreted as the total mass function of the collapsing dust cloud. According to collapse condition $\dot{R}(t,r) < 0$, i.e. the physical radius R is a monotonically decreasing function of t and ultimately becomes zero at constant value of r.

In the context of CCH, a realistic collapse scenario requires regular initial data (density, pressure and spacetime are free from singularity at initial epoch ($t = 0$)). Moreover, the initial data helps in determining the values of dynamical variables during collapse.[3–12] Thus, we consider the following set of initial data for our collapsing model

$$R(0,r) = r\,, \qquad R'(0,r) = 1\,, \qquad F(0,r) = \frac{r^3}{3}\,. \tag{16}$$

Using this set of initial data, Eq. (15) implies that

$$\rho^{(\mathrm{eff})}(0,r) = 1\,, \qquad P^{(\mathrm{eff})}(0,r) = K_{\mathrm{BD}}\rho^{(\mathrm{eff})}(0,r)\,, \qquad \dot{R}(0,r) = \frac{-r}{\sqrt{3}}\,,$$

which shows that $\rho^{(\mathrm{eff})}(0,r)$ and $P^{(\mathrm{eff})}(0,r)$ are smooth and analytic functions of r including $r = 0$. We use conservation equation, $T^{\mu\nu}_{(\mathrm{eff});\nu} = 0$, to find dynamical variables, $(\rho_{(\mathrm{eff})}, P^{(\mathrm{eff})}, F, R, R')$, at any value of t during collapse. The conservation relation yields

$$\dot{\rho}_{(\mathrm{eff})} = -\rho_{(\mathrm{eff})}(1 + K_{\mathrm{BD}})\left(\frac{2\dot{R}}{R} + \frac{\dot{R}'}{R'}\right), \qquad \dot{\rho}_m = -\rho_m\left(\frac{2\dot{R}}{R} + \frac{\dot{R}'}{R'}\right). \tag{17}$$

Integration of Eq. (17) gives

$$\rho_{(\mathrm{eff})}(t,r) = h(0,r)(R^{-2}\dot{R}^{-1})^{(1+K_{\mathrm{BD}})}\,, \qquad \rho_m(t,r) = k(0,r)R^{-2}\dot{R}^{-1}\,, \tag{18}$$

where

$$h(0,r) = (R^2(0,r)\dot{R}(0,r))^{(1+K_{\mathrm{BD}})}\,,$$

$$k(0,r) = \rho_m(0,r)(R^2(0,r)\dot{R}(0,r))^{(1+K_{\mathrm{BD}})}\,.$$

Using Eq. (18) in the equation of state parameter, it follows that

$$P_{(\mathrm{eff})} = K_{\mathrm{BD}}h(0,r)(R^{-2}\dot{R}^{-1})^{(1+K_{\mathrm{BD}})}\,, \tag{19}$$

and from Eqs. (15) and (18), we obtain

$$F = \frac{R^3 \rho_{(\text{eff})}}{3} = \frac{h(0,r) R^{(1-2K_{\text{BD}})} \dot{R}^{-(1+K_{\text{BD}})}}{3}\,. \tag{20}$$

Inserting this value of $F(t,r)$ in $\dot{R}^2$ (15), it follows that

$$\dot{R} = -\left[\frac{1}{3} h(0,r)\right]^{\frac{1}{3+K_{\text{BD}}}} R^{\frac{-2K_{\text{BD}}}{3+K_{\text{BD}}}}\,. \tag{21}$$

Integration of this equation implies that

$$R = \left(\frac{h(0,r)}{3}\right)^{\frac{1}{3(1+K_{\text{BD}})}} \left[-\frac{3(1+K_{\text{BD}})}{3+K_{\text{BD}}}(t - T(r))\right]^{\frac{3+K_{\text{BD}}}{3(1+K_{\text{BD}})}} \tag{22}$$

and its partial derivative with respect to r yields

$$R' = \frac{3+K_{\text{BD}}}{3(1+K_{\text{BD}})} A^{\frac{-2K_{\text{BD}}}{3(1+K_{\text{BD}})}} A'\,, \tag{23}$$

where

$$A = \left[-\frac{3(1+K_{\text{BD}})}{3+K_{\text{BD}}} \left(\frac{h(0,r)}{3}\right)^{\frac{1}{3+K_{\text{BD}}}} (t - T(r))\right].$$

Equation (22) represents the time dependence of the radius whereas $T(r)$ is an integration function which can be obtained by using initial epoch of collapse ($R(0,r) = r$) in (22) as

$$T(r) = \left(\frac{h(0,r)}{3}\right)^{\frac{-1}{3+K_{\text{BD}}}} \left[\left(\frac{3+K_{\text{BD}}}{3(1+K_{\text{BD}})}\right) r^{\frac{3(1+K_{\text{BD}})}{3+K_{\text{BD}}}}\right]. \tag{24}$$

The value of scalar field is found by integrating Eq. (14) as

$$\phi(t,r) = (\omega_{\text{BD}} + 1)^{\frac{1}{(\omega_{\text{BD}}+1)}} (cR + c')^{\frac{1}{(\omega_{\text{BD}}+1)}}\,, \tag{25}$$

where c and c' are constants of integration. Equations (25) and (11) yield

$$\begin{aligned}
\rho_\phi = {} & \frac{\omega_{\text{BD}}}{2(\omega_{\text{BD}}+1)}(cR + c')^{-2}\dot{R}^2 - \frac{\omega_{\text{BD}}}{2(\omega_{\text{BD}}+1)}(cR + c')^{-2}R'^2 \\
& - \frac{(\dot{R}'R^2)(cR + c')^{-1}\dot{R}}{R'R^2(\omega_{\text{BD}}+1)} + \left(\frac{R^2}{R'}\right)' \frac{(cR + c')^{-1}}{(\omega_{\text{BD}}+1)R^2} \\
& + \frac{(cR + c')^{-1}R''}{R'(\omega_{\text{BD}}+1)} - \frac{\omega_{\text{BD}}(cR + c')^{-2}R'}{(\omega_{\text{BD}}+1)^2} + \frac{U(\phi)}{2}\,.
\end{aligned}$$

The potential field is obtained by addition of radial and transversal components of the effective stress energy tensor along with (19) and (25) as

$$U(\phi) = \frac{\omega_{\rm BD}}{2}\frac{R^2\dot{\phi}^2}{\phi^2} + \left[\frac{\omega_{\rm BD}[R^2R'^2 - 1]}{2} + 1\right]\frac{\phi'}{\phi^2} + \left[1 - \frac{R^2}{R'}\right]\frac{\phi''}{\phi}$$
$$+ \frac{\phi'^2}{\phi} + \frac{R^2\ddot{\phi}}{\phi} + \left[\frac{R'}{R^2} - \frac{1}{R'}\right]\left(\frac{R^2}{R'}\right)'\frac{\phi'}{\phi}$$
$$- 2K_{\rm BD}h(0,r)R^{-2(1+K_{\rm BD})}\dot{R}^{-(1+K_{\rm BD})}\,. \qquad (26)$$

Equations (25) and (26) indicate that during the collapse of dust cloud, the behavior of scalar field and self-interacting potential is consistent with the physical radius $R(t,r)$.

In BD gravity, the only physically valid static spherically symmetric vacuum solution is the Schwarzschild solution.[42,43] Thus for a physically valid spherically symmetric collapse model, we describe the exterior region by a Schwarzschild metric as

$$ds^2 = -\left(1 - \frac{2F_s}{r_s}\right)dT^2 + \left(1 - \frac{2F_s}{r_s}\right)^{-1}dr_s^2 + r_s^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,. \qquad (27)$$

Here F_s and r_s represent the Schwarzschild mass and Schwarzschild radial coordinate, respectively. In BD theory, scalar field (a source of gravity) becomes constant in static or stationary region,[42,43] therefore the matching of interior and exterior region requires $\phi = \phi_\Sigma =$ const. at the hypersurface boundary Σ. Using Darmois junction conditions at the Σ,[3–12] we obtain $r_s = R(t, r_c)$ and $F(t, r_c) = 2F_s$. The relation $F(t, r_c) = 2F_s$ describes the total Schwarzschild mass enclosed within the dust ball at $r = r_c$. We check the validity of weak energy condition throughout the collapse[3–12] such as

$$\rho_{\rm (eff)} \geq 0\,, \qquad \rho_{\rm (eff)} + P_{\rm (eff)} \geq 0\,.$$

Using Eqs. (18) and (19) in the above inequalities, it follows that

$$h(0,r)(R^{-2}\dot{R}^{-1})^{(1+K_{\rm BD})} \geq 0\,, \qquad (28)$$

$$(1 + K_{\rm BD})h(0,r)(R^{-2}\dot{R}^{-1})^{(1+K_{\rm BD})} \geq 0\,, \qquad (29)$$

where $(1+K_{\rm BD})$ is an even valued nonnegative number so that $\dot{R}^{-(1+K_{\rm BD})}$ remains positive in the above defined inequalities. Moreover, from Eqs. (15) and (23), $F' \geq 0$, and $R' \geq 0$, respectively. Since $(1 + K_{\rm BD})$ is taken to be an even number, therefore only that class of solutions is consistent with weak energy conditions for which $K_{\rm BD}$ is nonnegative odd number.

3. Singularities

Singularities are the points where all physical quantities like curvature invariants, density that measure gravitational field diverge.[44] In order to find singular points, we use the Kretschmann scalar (a curvature invariant), $\Re = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ which takes the form

$$\Re = 12\frac{F'^2}{R^4R'^2} - 32\frac{FF'}{R^5R'} + 48\frac{F^2}{R^6}\,. \tag{30}$$

Equations (15) and (30) indicate that both density and curvature invariant diverge whenever $R = 0$ or $R' = 0$. This leads to two singularities: shell focusing singularity at $R = 0$ and shell crossing singularity when $R > 0$, $F' > 0$ and $R' = 0$. According to CCH, the shell crossing singularity is not a physical singularity and can be avoided by considering $R' \neq 0$.[3–12] Shell focusing singularities are the physical singularities that arise when all the spherical shells of cloud are crushed to zero volume. There are two types of shell focusing singularities, central at $R(t, r = 0) = 0$ and non-central at $R(t, r \neq 0) = 0$. Since Eq. (22) shows that at $t = T(r)$, physical radius of shell goes to zero, so $T(r)$ in Eq. (24) represents the time of singular epoch or time of shell focusing singularity.

The nature of singularity can be found by evaluating the time formation of apparent horizon. The apparent horizon denotes the boundary of trapped null surfaces of the spacetime[45] for which the spherically symmetric spacetime is

$$R^{,l}R_{,l} = g^{kl}R_{,k}R_{,l} = 0\,. \tag{31}$$

Equations (10) and (31) imply that $R = F$ which indicates the equivalence of physical radius and mass at apparent horizon. Using $R(t_{\text{ap}}, r) = F(t_{\text{ap}}, r)$ in Eq. (22), we obtain

$$t_{\text{ap}} = T(r) - \left(\frac{3 + K_{\text{BD}}}{3(1 + K_{\text{BD}})}\right)\left(F(t_{\text{ap}}, r)^{\frac{3(1+K_{\text{BD}})}{3+K_{\text{BD}}}}\right)\left(\frac{h(0,r)}{3}\right)^{\frac{-1}{3+K_{\text{BD}}}}. \tag{32}$$

This is the time formation of apparent horizon which implies that $t_{\text{ap}} < T(r)$, i.e. apparent horizon is formed earlier than singularity at $R(T(r), r) = 0$, leading to hidden singularity, i.e. BH. Furthermore, $R(t_{\text{ap}}, 0) = F(t_{\text{ap}}, 0)$ implies that

$$t_{\text{ap}} = T(0) - \left(\frac{3 + K_{\text{BD}}}{3(1 + K_{\text{BD}})}\right)\left(F(t_{\text{ap}}, 0)^{\frac{3(1+K_{\text{BD}})}{3+K_{\text{BD}}}}\right)\left(\frac{h(0,r)}{3}\right)^{\frac{-1}{3+K_{\text{BD}}}}. \tag{33}$$

Since according to the energy conditions, the value of K_{BD} is nonnegative odd number, so the second term in the above equation becomes zero and hence $t_{\text{ap}} = T(0)$, which indicates the possibility of naked singularity at symmetric center.

In order to confirm the nakedness of central singularity, we analyze the existence of outgoing radial null geodesics that are emanating from central singularity.[8,40] Any radial null geodesic traveling outwards from the center satisfies

$$\frac{dt}{dr} = R'\,, \qquad \theta = \text{const.}\,, \qquad \phi = \text{const.}\,, \tag{34}$$

which implies that

$$\frac{dR}{dr} = R'(1 + \dot{R})\,. \tag{35}$$

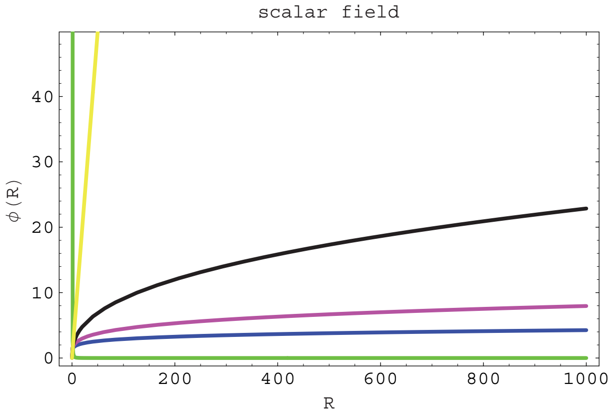

Fig. 1. (color online) Plot of BD scalar field vs. $R(t,r)$ for $\omega_{\rm BD}=5$ (blue), $\omega_{\rm BD}=3$ (purple), $\omega_{\rm BD}=\frac{-3}{2}$ (green), $\omega_{\rm BD}=\frac{3}{2}$ (black) and $\omega_{\rm BD}=0$ (yellow) with $c=1$, $c'=0$.

To investigate the existence of radial null geodesics at the symmetric center, we consider a new variable $v=r^{\beta}$ such that $\beta>1$ and $\frac{R'}{r^{\beta-1}}$ remains a unique finite term at $r\to 0$. In this case, Eq. (35) yields

$$\frac{dR}{dv}=\frac{R'}{\beta r^{\beta-1}}(1+\dot{R})\,. \tag{36}$$

If there exist outgoing radial null geodesics terminating in the past at the singularity with a definite tangent, we have $\frac{dR}{dv}>0$ when $R\to 0$, $r\to 0$. Equations (21) and (36) imply that

$$\lim_{R\to 0,r\to 0}\left(\frac{dR}{dv}\right)=-\infty$$

and hence there is no radial null geodesics emanating from the symmetric center which leads to hidden central singularity. Thus in both cases, non-central and central shell focusing singularities are hidden within event horizon forming a BH. We plot the scalar field (25) for different values of $\omega_{\rm BD}$ and $R(t,r)$ near singularities as shown in Fig. 1.

4. Conclusion

In this paper, we have studied gravitational collapse of inhomogeneous dust cloud in BD theory with perfect fluid by taking suitable initial data. The equations of continuity have been used to obtain values of the effective density, effective pressure and matter density. We have found a class of collapse solutions of flat LTB model that depend on the parameters $K_{\rm BD}$, $\omega_{\rm BD}$ and the physical radius $R(t,r)$.

We have also investigated the time formation of apparent horizon and singular epoch with the help of Eq. (22) which depends on r rather than constant contrary to the Friedmann collapse case. We have also checked the validity of weak energy conditions (28) and (29) for LTB model. The shell focusing and shell crossing singularities are explored and the end state of dust collapse is discussed which is consistent with CCH. Finally, the scalar field is shown as a function of physical radius and its behavior is studied near singularities for different values of $\omega_{\rm BD}$ graphically. The potential field (33) depending on physical radius $R(t, r)$ is obtained with the help of effective energy–momentum tensor. It is mentioned here that collapse solutions of flat LTB model do not allow the formation of naked singularities in the presence of pressure due to scalar field. Hence the final fate of a dust turns out to be a BH in BD gravity but such singularities cannot be formed in GR.[3–12]

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