An Intuitionistic 2-Tuple Linguistic Information Model and Aggregation Operators

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Dealing with uncertainty is always a challenging problem, and different tools have been proposed to deal with it. Fuzzy sets was presented to manage situations in which experts have some membership value to assess an alternative. The fuzzy linguistic approach has been applied successfully to many problems. The linguistic information expressed by means of 2-tuples, which were composed by a linguistic term and a numeric value assessed in $[-0.5, 0.5)$. Linguistic values was used to assess an alternative and variable in qualitative settings. Intuitionistic fuzzy sets were presented to manage situations in which experts have some membership and nonmembership value to assess an alternative. In this paper, the concept of an I2LI model is developed to provide a linguistic and computational basis to manage the situations in which experts assess an alternative in possible and impossible linguistic variable and their translation parameter. A method to solve the group decision making problem based on intuitionistic 2-tuple linguistic information (I2LI) by the group of experts is formulated. Some operational laws on I2LI are introduced. Based on these laws, new aggregation operators are introduced to aggregate the collective opinion of decision makers. An illustrative example is given to show the practicality and feasibility of our proposed aggregation operators and group decision making method. © 2015 Wiley Periodicals, Inc.

1. INTRODUCTION

Atanassov[1] gave the notion of intuitionistic fuzzy set (IFS), which is an extension of Zadeh's[2] fuzzy set. IFS has received more attention and has been applied in the field of decision making.[3–5] The concept of interval-valued IFS was also introduced by Atanassov[6] and Gragov, as a generalization of IFS. The basic characteristic of the interval-valued IFS is that the values of its membership and nonmembership functions are intervals rather than exact numbers. Tan[7] gave a novel method for multiple attribute decision making (MADM) based on interval-valued IFS. Xu[8] defined geometric aggregation operators, such as the interval-valued intuitionistic

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INTERNATIONAL JOURNAL OF INTELLIGENT SYSTEMS, VOL. 31, 569–592 (2016)

View this article online at wileyonlinelibrary.com. • DOI 10.1002/int.21795

fuzzy weighted geometric averaging operator and the interval-valued intuitionistic fuzzy ordered weighted geometric averaging operator and showed applications of these operators to multiple attribute group decision making (MAGDM) with interval-valued intuitionistic fuzzy information are given. Wei[9] applied interval-valued intuitionistic fuzzy weighted geometric aggregation functions in dealing with dynamic MADM where all the attribute values are expressed in intuitionistic fuzzy numbers or interval-valued intuitionistic fuzzy numbers.

The attributes are assumed to be independent of each other in all above-mentioned operators.[10] But in many real-world decision-making problems, the attributes of the decision problem are correlated. The Choquet integral introduced by Choquet[11] as a useful tool to model the correlation or inter-dependence, and this integral has been applied in many decision-making methods. Meng et al.[12] proposed two aggregation operators called the arithmetical interval-valued intuitionistic fuzzy generalized λ-Shapley Choquet operator and the geometric interval-valued intuitionistic fuzzy generalized λ-Shapley Choquet operator. They used these operators to solve multiple-criteria group decision making problem. Aggregation of decision makers' (DM) opinions plays an important role in group decision making problems to perform evaluation process.[13,14] Group decision making involves weighted aggregation of all individual decisions to get a single collective decision. In Ref. 15, aggregation operator of intuitionistic fuzzy group decision making is proposed with the weights of DM. The weights of DM play an important role in the process of aggregation. In Ref. 7, the Choquet integral based aggregation operator for interval-valued IFS is studied. In Ref. 16, the Choquet integral has been used for the aggregation of IFS. The intuitionistic fuzzy Choquet integral and the induced Choquet-ordered averaging operator were proposed by Tan.[17,18] Many aggregation operators for the IFS were developed by Xu in Ref. 19, such as intuitionistic fuzzy correlated geometric operators, the interval-valued intuitionistic fuzzy correlated geometric operator, the intuitionistic fuzzy correlated averaging operator, and the interval-valued intuitionistic fuzzy correlated averaging operator. The Choquet integral based aggregation of fuzzy numbers and fuzzy extension was defined by Meyer and Roubens.[20] The Choquet integral based aggregation operator for the 2-tuple linguistic information was formulated by Yang and Chen in Ref. 21.

The fuzzy linguistic approach[22] has provided good results in many fields and applications by experts in problems whose nature is rather qualitative.[23–25] But in some situations, the fuzzy linguistic approach is also limited for computational processes, which are called processes of computing with words.[26,27] The 2-tuple fuzzy linguistic representation model was developed in Ref. 28 to avoid the loss of information in the process of linguistic information. This information model has been applied and studied in the decision-making problems, and to aggregate this information many aggregation operators have been developed in Refs. 29, 30 and 31. The arithmetic, arithmetic weighted, ordered weighted, and extended weighted averaging operators were developed to aggregate the 2-tuple linguistic information.[31–33] Furthermore, the extended geometric mean aggregation operator, the extended arithmetic averaging aggregation operator, the extended ordered weighted averaging aggregation operator, and the extended ordered weighted geometric aggregation operator were presented in Ref. 30. The extended 2-tuple ordered weighted geometric operator and the extended 2-tuple weighted geometric operator have been defined in Ref. 34. Recently, Beg and Rashid[35] introduced the notion of interval valued 2-tuple fuzzy linguistic information and aggregation operators. Furthermore, these operators were used in the MADM method. Zhang et al.[36] studied the transitivity of 2-tuple intuitionistic fuzzy linguistic preference relations to solve group decision making problems.

In this paper, we propose notion of intuitionistic 2-tuple linguistic information (I2LI) model and some Choquet integral based operators to aggregate the I2LI. This paper is organized as follows: Some basic notions of 2-tuple linguistic information are given in Section 2. In Section 3, the concept of intuitionistic 2-tuple linguistic representation model is proposed with symbolic translation of linguistic arguments. In Section 4, an intuitionistic 2-tuple correlated averaging operator is proposed. In Section 5, the generalized intuitionistic 2-tuple correlated averaging operator is defined. Cases of these new operators and the properties of these new aggregation operators are studied in their respective sections. The MADM method is proposed in Section 6. In Section 7, an illustrative example is given to show the practicality and feasibility of our proposed method. The conclusion of the paper is given in the last section.

2. PRELIMINARIES

Let X be a universe of discourse, a fuzzy set[2] in X is an expression A given by $A = \{\langle x, t_A(x)\rangle \,|x \in X\}$, where $t_A : X \to [0, 1]$ is a membership function, which characterizes the degree of membership of the element x to the set A. The main characteristic of fuzzy sets is that the membership function assigns to each element x in a universe of discourse X a membership degree in interval [0, 1], and the nonmembership degree equals one minus the membership degree, i.e. this single membership degree combines the evidence for x and the evidence against x. In real applications, however, the information of an object corresponding to a fuzzy concept may be incomplete, i.e., the sum of the membership degree and the nonmembership degree of an element in a universe corresponding to a fuzzy concept may be less than one. In fuzzy set theory, there is no means to incorporate the lack of knowledge with the membership degrees. In 1986, Atanassov[1] generalized the concept of fuzzy set and defined the concept of IFS.

Definition 2.1. *Let $X = \{x_1, x_2, \ldots\}$ be a universe of discourse, an IFS in X is an expression A given by $A = \{(x, t_A(x), f_A(x))|x \in X\}$, where $t_A : X \to [0, 1]$, $f_A : X \to [0, 1]$ with the condition: $0 \leq t_A(x) + f_A(x) \leq 1$, for all x in X. The numbers $t_A(x)$ and $f_A(x)$ represent the degree of membership and the degree of nonmembership of the element x in the set A, respectively.*

For each IFS A in X, if $\pi_A(x) = 1 - (t_A(x) + f_A(x))$, $\forall x \in X$. Then $\pi_A(x)$ is called the degree of indeterminacy of x to A.

If $\pi_A(x) = 1 - t_A(x) + f_A(x) = 0, \qquad \forall x \in X$ then the IFS A reduces to a fuzzy set.

Definition 2.2. *[37] A fuzzy measure* $\mu : P(X) \to [0, 1]$ *satisfying the following three axioms:*

(1) $\mu(\phi) = 0,\ \mu(X) = 1$;
(2) $B \subseteq C$ *implies* $\mu(B) \leq \mu(C)$, *for all* $B, C \subseteq X$;
(3) $\mu(B \cup C) = \mu(B) + \mu(C) + \lambda\mu(B)\mu(C)$ *for all* $B, C \subseteq X$ *and* $B \cap C = \phi$, *where* $\lambda \in (-1, +\infty)$.

By parameter λ the interaction between criteria can be represented.

If X is a finite set, then $\bigcup_{i=1}^{n} x_i = X$. The λ-fuzzy measure μ satisfies the following Equation 2.1:

$$\mu(X) = \mu\left(\bigcup_{i=1}^{n} x_i\right) = \begin{cases} \dfrac{1}{\lambda}\left\{\prod_{i=1}^{n}[1 + \lambda\mu(x_i)] - 1\right\} & if \lambda \neq 0, \\ \sum_{i=1}^{n} \mu(x_i) & if \lambda = 0, \end{cases} \tag{2.1}$$

where $x_i \cap x_j = \phi$ for all $i, j = 1, 2, \ldots, n$ and $i \neq j$. The number $\mu(x_i)$ for a subset with a single element $\{x_i\}$ is called a fuzzy density.

For any subset A of X,

$$\mu(A) = \begin{cases} \dfrac{1}{\lambda}\left\{\prod_{x_i \in A}[1 + \lambda\mu(x_i)] - 1\right\} & if \lambda \neq 0, \\ \sum_{x_i \in A} \mu(x_i) & if \lambda = 0. \end{cases} \tag{2.2}$$

Based on Equation 2.1, the value of λ can be uniquely determined from $\mu(X) = 1$, which is equivalent to solving

$$1 = \frac{1}{\lambda}\left\{\prod_{i=1}^{n}[1 + \lambda\mu(x_i)] - 1\right\}. \tag{2.3}$$

If all the elements of B in X are independent, we have

$$\mu(B) = \sum_{x_i \in B} \mu(x_i),\ \text{for any } B \subseteq X. \tag{2.4}$$

In Ref. 32, the linguistic information is expressed by means of 2-tuples, which were composed by a linguistic term and a numeric value assessed in $[-0.5, 0.5)$.

Definition 2.3. *[38] Suppose that* $S = \{s_i | i = 1, \ldots, t\}$ *is a finite and totally ordered discrete term set, where* s_i *represents a possible linguistic term for a linguistic variable.*

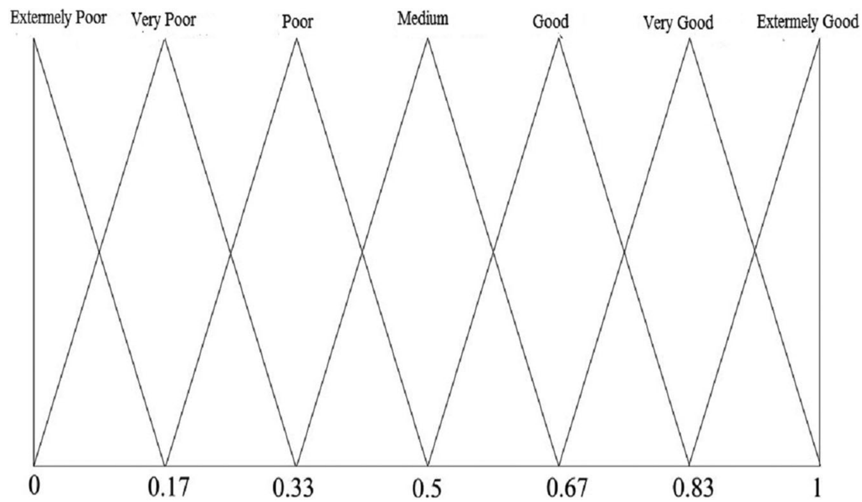

Figure 1. Set of seven terms with its semantics.

EXAMPLE *2.4. Let S be a linguistic term set (Figure 1), $S = \{s_1 =$ extremely poor (EP), $s_2 =$ very poor (VP), $s_3 =$ poor (P), $s_4 =$ medium (M), $s_5 =$ good (G), $s_6 =$ very good (VG), $s_7 =$ extremely good (EG)$\}$.*

DEFINITION 2.5. *[38] The linguistic term set, $S = \{s_i | i = 1, \ldots, t\}$ satisfies the following properties:*

(1) The set is ordered: $s_i \geq s_j$, if $i \geq j$;
(2) max operator: $\max(s_i, s_j) = s_i$, if $i \geq j$;
(3) min operator: $\min(s_i, s_j) = s_i$, if $i \leq j$.

DEFINITION 2.6. *[32] The 2-tuple (s_i, α_i) is used to represent the linguistic information, where s_i is a linguistic label from a predefined linguistic term set S and $\alpha_i \in [-0.5, 0.5)$ is representing the value of the possible symbolic translation of s_i.*

3. INTUITIONISTIC 2-TUPLE FUZZY LINGUISTIC REPRESENTATION MODEL

DEFINITION 3.1. *Let X be a universe of discourse, and $S = \{s_1, \ldots, s_t\}$ be a linguistic term set, an intuitionistic linguistic term set in X is an expression A given by $A = \{(x, h(x), h'(x)) | x \in X\}$, where $h, h' : X \to S$, such that $h(x) = s_i$ and $h'(x) = s_j$ with the condition: $2 \leq i + j \leq t + 1$, for all x in X. The numbers $h(x)$ and $h'(x)$ represent the degree of membership and nonmembership of the element x in the set A, respectively.*

DEFINITION 3.2. *Let (s_i, s_j) be an element of intuitionistic linguistic term set in X, an intuitionistic 2-tuple linguistic model is $((s_i, \alpha), (s_j, \eta))$, where s_i, s_j are* *linguistic terms, and* $\alpha, \eta \in [-0.5, 0.5]$ *are numeric values representing the symbolic translation, respectively.*

Together with this representation model, we shall present a computational technique to deal with an intuitionistic 2-tuple linguistic model without loss of information.

DEFINITION 3.3. *Let* S *be a linguistic term set and* $((s_i, \alpha), (s_j, \eta))$ *be an intuitionistic 2-tuple. The function* ∇ *from an intuitionistic 2-tuple* $((s_i, \alpha), (s_j, \eta))$ *to an order pair numerical values* $(\beta, \zeta) \in [-0.5, t+0.5) \times [-0.5, t+0.5) \subset \mathbb{R} \times \mathbb{R}$ *defined as follows:*

$$\nabla : (S \times [-0.5, 0.5)) \times (S \times [-0.5, 0.5)) \rightarrow [0.5, t+0.5) \times [0.5, t+0.5),$$

such that

$$\nabla((s_i, \alpha), (s_j, \eta)) = (i + \alpha, j + \eta) = (\beta, \zeta).$$

DEFINITION 3.4. *Let* (β, ζ) *be an intuitionistic order pair of the result of an aggregation of the intuitionistic 2-tuple linguistic model in a linguistic term set* S*, i.e., the result of a symbolic aggregation operation,* $\beta, \zeta \in [0.5, t+0.5)$ *with the condition* $1 \leq \beta + \zeta < t + 2$*, where* t *being the cardinality of* S.

DEFINITION 3.5. *Let* S *be a linguistic term set and* (β, ζ) *be an intuitionistic order pair of two numbers representing the aggregation results of linguistic symbolic. The function*

$$\Delta : [0.5, t+0.5) \times [0.5, t+0.5) \rightarrow (S \times [-0.5, 0.5)) \times (S \times [-0.5, 0.5)),$$

used to obtain the I2LI equivalent to (β, ζ) *is defined as*

$$\Delta(\beta, \zeta) = ((s_i, \alpha), (s_j, \eta))$$

where $i = \text{round}(\beta)$, $j = \text{round}(\eta)$ *, and* $\alpha = \beta - i$, $\eta = \zeta - j$, s_i *and* s_j *has the closest index label to* β, ζ *and* α, η *are the value of the symbolic translations of* s_i *and* s_j*, respectively.*

REMARK 3.6. *Composition of* ∇ *and* Δ *is an identity mapping, i.e.*

$$\Delta\left(\nabla((s_i, \alpha), (s_j, \eta))\right) = ((s_i, \alpha), (s_j, \eta)).$$

DEFINITION 3.7. *Let* $((s_i, \alpha_i), (s_j, \eta_j))$ *and* $((s_m, \alpha_m), (s_n, \eta_n))$ *be two intuitionistic 2-tuples, they have the following properties:*

(1) If $s_i < s_m$ *then* $((s_i, \alpha_i), (s_j, \eta_j)) < ((s_m, \alpha_m), (s_n, \eta_n))$,
(2) If $s_i = s_m$*, and;*
  (i) $\alpha_i < \alpha_m$*, then* $((s_i, \alpha_i), (s_j, \eta_j)) < ((s_m, \alpha_m), (s_n, \eta_n))$,
  (ii) $\alpha_i = \alpha_m$*, and;*
    (a) $s_n < s_j$*, then* $((s_i, \alpha_i), (s_j, \eta_j)) < ((s_m, \alpha_m), (s_n, \eta_n))$,

(b) $s_j = s_n$, and;
(I) $\eta_n < \eta_j$, then $((s_i, \alpha_i), (s_j, \eta_j)) < ((s_m, \alpha_m), (s_n, \eta_n))$,
(II) $\eta_n = \eta_j$, then $((s_i, \alpha_i), (s_j, \eta_j)) = ((s_m, \alpha_m), (s_n, \eta_n))$.

DEFINITION 3.8. *Let (β_i, ζ_i) and (β_j, ζ_j) be two intuitionistic order pairs, they have the following properties:*

(1) If $\beta_i < \beta_j$ then $(\beta_i, \zeta_i) < (\beta_j, \zeta_j)$,
(2) If $\beta_i = \beta_j$, and;
(i) $\zeta_j < \zeta_i$ then $(\beta_i, \zeta_i) < (\beta_j, \zeta_j)$,
(ii) $\zeta_j = \zeta_i$ then $(\beta_i, \zeta_i) = (\beta_j, \zeta_j)$.

DEFINITION 3.9. *Let (β, ζ) be an intuitionistic order and $0 \leq k \leq 1$; the product of this constant k with (β, ζ) is defined as*

$$k(\beta, \zeta) = (k\beta, k\zeta)$$

DEFINITION 3.10. *Let (β_i, ζ_i), (β_j, ζ_j) be two intuitionistic order pairs and $0 \leq k_1, k_2 \leq 1$ with $0 \leq k_1 + k_2 \leq 1$, then*

(1) $k_1(\beta_i, \zeta_i) \cdot k_2(\beta_j, \zeta_j) = (k_1\beta_i \cdot k_2\beta_j, k_1\zeta_i \cdot k_2\zeta_j)$
(2) $(\beta_i, \zeta_i)^\tau = (\beta_i{}^\tau, \zeta_i{}^\tau)$ for all $0 \leq \tau \leq 1$
(3) $k_1(\beta_i, \zeta_i) + k_2(\beta_j, \zeta_j) = (k_1\beta_i + k_2\beta_j, k_1\zeta_i + k_2\zeta_j)$

LEMMA 3.11. *If $((s_i, \alpha_i), (s_j, \eta_j)) \leq ((s_m, \alpha_m), (s_n, \eta_n))$. Then $\nabla((s_i, \alpha_i), (s_j, \eta_j)) \leq \nabla((s_m, \alpha_m), (s_n, \eta_n))$.*

Proof. To hold $((s_i, \alpha_i), (s_j, \eta_j)) \leq ((s_m, \alpha_m), (s_n, \eta_n))$ the following cases are possible.

Case 1. If $s_i < s_m$, then $i < m$.
Thus $i + \alpha_i < m + \alpha_m$ for any $\alpha_i, \alpha_m \in [-0.5, 0.5)$.
This implies that $\beta_i < \beta_m$.
So, $(\beta_i, \zeta_j) < (\beta_m, \zeta_n)$.
This yields that $\nabla((s_i, \alpha_i), (s_j, \eta_j)) < \nabla((s_m, \alpha_m), (s_n, \eta_n))$.

Case 2. If $s_i = s_m$ and $\alpha_i < \alpha_m$, then $i = m$
This shows that $i + \alpha_i < m + \alpha_m$ where $\alpha_i, \alpha_m \in [-0.5, 0.5)$.
Therefore, $\beta_i < \beta_m$, which implies that $(\beta_i, \zeta_j) < (\beta_m, \zeta_n)$.
Thus, $\nabla((s_i, \alpha_i), (s_j, \eta_j)) < \nabla((s_m, \alpha_m), (s_n, \eta_n))$.

Case 3. If $s_i = s_m$, $\alpha_i = \alpha_m$, and $s_n < s_j$, then $i = m$ and $n < j$.
So, $i + \alpha_i = m + \alpha_m$ and $n + \eta_n < j + \eta_j$ for any $\eta_n, \eta_j \in [-0.5, 0.5)$.
This implies that $\beta_i = \beta_m$ and $\zeta_n < \zeta_j$, which forces that $(\beta_i, \zeta_j) < (\beta_m, \zeta_n)$.
This yields that $\nabla((s_i, \alpha_i), (s_j, \eta_j)) < \nabla((s_m, \alpha_m), (s_n, \eta_n))$.

Case 4. If $s_i = s_m$, $\alpha_i = \alpha_m$, $s_n = s_j$, and $\eta_n < \eta_j$, then $i = m$ and $n = j$.
It can be easily seen that $i + \alpha_i = m + \alpha_m$ and $n + \eta_n < j + \eta_j$, where $\eta_n, \eta_j \in [-0.5, 0.5)$.
This shows that $\beta_i = \beta_m$ and $\zeta_n < \zeta_j$, which follow that $(\beta_i, \zeta_j) < (\beta_m, \zeta_n)$.
The net effect is $\nabla((s_i, \alpha_i), (s_j, \eta_j)) < \nabla((s_m, \alpha_m), (s_n, \eta_n))$.

Case 5. If $s_i = s_m$, $\alpha_i = \alpha_m$, $s_n = s_j$ and $\eta_n = \eta_j$, then $i = m$ and $n = j$.
It follows that $i + \alpha_i = m + \alpha_m$ and $n + \eta_n = j + \eta_j$.
Thus, $\beta_i = \beta_m$ and $\zeta_n = \zeta_j$.

Therefore, $(\beta_i, \zeta_j) = (\beta_m, \zeta_n)$.
This yields $\nabla((s_i, \alpha_i), (s_j, \eta_j)) = \nabla((s_m, \alpha_m), (s_n, \eta_n))$.

□

LEMMA 3.12. *If* $(\beta_i, \zeta_j) \leq (\beta_m, \zeta_n)$. *Then* $\Delta(\beta_i, \zeta_j) \leq \Delta(\beta_m, \zeta_n)$.

Proof. To hold $(\beta_i, \zeta_j) \leq (\beta_m, \zeta_n)$, the following cases are possible.

Case 1. If $\beta_i < \beta_m$
This implies that $\text{round}(\beta_i) < \text{round}(\beta_m)$, which forces that $i \leq m$ where $i = \text{round}(\beta_i)$ and $j = \text{round}(\beta_m)$.

- If $i < m$.
  Implies $s_i < s_j$, which follows that $\Delta(\beta_i, \zeta_j) < \Delta(\beta_m, \zeta_n)$.
- If $i = m$.
  It implies that $\beta_i - i < \beta_m - m$, which forces that $\alpha_i < \alpha_m$ where $\alpha_i = \beta_i - i$ and $\alpha_m = \beta_m - m$.
  This yields $\Delta(\beta_i, \zeta_j) < \Delta(\beta_m, \zeta_n)$.

Case 2. If $\beta_i = \beta_m$ and $\zeta_n < \zeta_j$
Implies $\text{round}(\beta_i) = \text{round}(\beta_m)$ and $\text{round}(\zeta_n) \leq \text{round}(\zeta_j)$, which further shows that $i = m$ and $n \leq j$, where $i = \text{round}(\beta_i)$, $j = \text{round}(\beta_m)$, $n = \text{round}(\zeta_n)$ and $j = \text{round}(\zeta_j)$.
Hence $\alpha_i = \alpha_m$ and $\eta_n < \eta_j$, where $\alpha_i = \beta_i - i, \alpha_m = \beta_m - m, \eta_n = \zeta_n - n, \eta_j = \zeta_j - j$.

- If $n < j$, which implies that $s_n < s_j$
  This shows that $\Delta(\beta_i, \zeta_j) < \Delta(\beta_m, \zeta_n)$.
- If $n = j$
  This implies that $\zeta_n - n < \zeta_j - j$, which follows that $\eta_n < \eta_j$
  The net effect is $\Delta(\beta_i, \zeta_j) < \Delta(\beta_m, \zeta_n)$.

Case 3. If $\beta_i = \beta_m$ and $\zeta_n = \zeta_j$
This implies that $\text{round}(\beta_i) = \text{round}(\beta_m)$ and $\text{round}(\zeta_n) = \text{round}(\zeta_j)$.
It follows that $i = m$ and $n = j$, where $i = \text{round}(\beta_i)$, $j = \text{round}(\beta_m)$, $n = \text{round}(\zeta_n)$ and $j = \text{round}(\zeta_j)$
Therefore, $\alpha_i = \alpha_m$ and $\eta_n = \eta_j$, where $\alpha_i = \beta_i - i, \alpha_m = \beta_m - m, \eta_n = \zeta_n - n, \eta_j = \zeta_j - j$.
Hence, $\Delta(\beta_i, \zeta_j) = \Delta(\beta_m, \zeta_n)$.

□

4. INTUITIONISTIC 2-TUPLE CORRELATED AVERAGING OPERATOR

The Choquet integral based new aggregation operators with correlative weights is proposed here. Furthermore, some properties and remarks of these operators are obtained.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the set of the attributes and $\mu(x_i)$ $(i = 1, 2, \ldots, n)$ be the weight of the elements $x_i \in X$, where μ is a λ-fuzzy measure.

DEFINITION 4.1. *Let* $((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n'))$ *be intuitionistic 2-tuple linguistic arguments (I2LA),* X *be the set of attributes and* μ *on* X, *then the intuitionistic 2-tuple correlated averaging (I2TCA) operator is defined* *as follows:*

$$I2TCA_{\mu}(((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n)))$$
$$=\Delta\left(\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))\right),$$

here $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ *is a permutation of* $(1,2,\ldots,n)$ *such that* $((r_{\sigma(1)},\alpha_{\sigma(1)}),(r'_{\sigma(1)},\quad\alpha'_{\sigma(1)}))\geq((r_{\sigma(2)},\alpha_{\sigma(2)}),(r'_{\sigma(2)},\alpha'_{\sigma(2)}))\geq\cdots\geq((r_{\sigma(n)},\alpha_{\sigma(n)}),(r'_{\sigma(n)},\alpha'_{\sigma(n)}))$, *where* $x_{\sigma(i)}$ *is the attribute corresponding to* $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$, $H_{\sigma(i)}=\{x_{\sigma(k)}|k\leq i\}$ *for* $i\geq 1$, $H_{\sigma(0)}=\phi$.

Now we consider some special cases of the I2TCA operator. Let $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$ $(i=1,2,\ldots,n)$ be I2LA and μ be a fuzzy measure on X.

Case I. If $\mu(H)=1$, for any $H\in P(X)$, then

$$I2TCA_{\mu}(((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n)))$$
$$=((r_{\sigma(1)},\alpha_{\sigma(1)}),(r'_{\sigma(1)},\alpha'_{\sigma(1)})).$$

Case II. If $\mu(H)=0$ for any $H\in P(X)$ and $H\neq X$, then

$$I2TCA_{\mu}(((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n)))$$
$$=((r_{\sigma(n)},\alpha_{\sigma(n)}),(r'_{\sigma(n)},\alpha'_{\sigma(n)})).$$

Case III. For any $A,B\in P(X)$ such that $|A|=|B|$, where $|A|$ and $|B|$ are the number of the elements in A and B, respectively, if $\mu(A)=\mu(B)$ and $\mu(H_{\sigma(i)})=\frac{i}{n}$, $1\leq i\leq n$, then

$$I2TCA_{\mu}(((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n)))$$
$$=\Delta\left(\sum_{i=1}^{n}\frac{1}{n}\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))\right)$$
$$=\Delta\left(\sum_{i=1}^{n}\frac{1}{n}\nabla((r_i,\alpha_i),(r'_i,\alpha'_i))\right).$$

Case IV. If condition (2.4) holds, then

$$\mu(x_{\sigma(i)})=\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}),\ i=1,2,\ldots,n. \tag{4.1}$$

In this case, the I2TCA operator reduces to the following intuitionistic 2-tuple weighted averaging (I2TWA) operator:

$$I2TWA_{\mu}(((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n)))$$
$$=\Delta\left(\sum_{i=1}^{n}\mu(x_i)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))\right).$$

Case V. For $H \in P(X)$, let

$$w_i = \mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)}), i = 1, 2, \ldots, n, \tag{4.2}$$

$\sum_{i=1}^{n} w_i = 1$ and $w = (w_1, w_2, \ldots, w_n)^T$.
If

$$\mu(H) = \sum_{i=1}^{|H|} w_i, \tag{4.3}$$

where $|H|$ is the number of the elements in H, then, the I2TCA operator reduces to the intuitionistic 2-tuple ordered weighted averaging (I2TOWA) operator:

$$I2TOWA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$= \Delta\left(\sum_{i=1}^{n} w_i \nabla\left(r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)})\right)\right).$$

Case VI. For $H \in P(X)$, let M be a function $M : [0, 1] \to [0, 1]$, which has the following three properties:
(i) $M(0) = 0$;
(ii) $M(1) = 1$;
(iii) $M(x) \geq M(y)$ for $x > y$; and

$$w_i = M\left(\sum_{j \leq i} \mu(x_{\sigma(j)})\right) - M\left(\sum_{j < i} \mu(x_{\sigma(j)})\right), i = 1, 2, \ldots, n, \tag{4.4}$$

where $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \geq 0$, $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} w_i = 1$.
If

$$\mu(H) = M\left(\sum_{x_i \in H} \mu(x_i)\right), \tag{4.5}$$

then the I2TCA operator reduces to the following intuitionistic 2-tuple weighted ordered weighted averaging (I2TWOWA) operator:

$$I2TWOWA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$= \Delta\left(\sum_{i=1}^{n} w_i \nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)}))\right).$$

Next, we further investigate some properties of the I2TCA operator.

THEOREM 4.2. *(Idempotent). Let* $((r_i, \alpha_i), (r_i', \alpha_i'))$ $(i = 1, 2, \ldots, n)$ *be I2LA and* μ *on* X. *If all the* $((r_i, \alpha_i), (r_i', \alpha_i'))$ *are equal, that is, for all* i, $((r_i, \alpha_i), (r_i', \alpha_i')) =$

$((r, \alpha), (r', \alpha'))$, *then*

$$I2TCA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$= ((r, \alpha), (r', \alpha')).$$

Proof. We have

$$I2TCA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$= \Delta\left(\sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right) \nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)}))\right).$$

As we know $((r_i, \alpha_i), (r_i', \alpha_i')) = ((r, \alpha), (r', \alpha'))$ for all i.

$$I2TCA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$= \Delta\left(\sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right) \nabla((r, \alpha), (r', \alpha'))\right)$$
$$= \Delta\left(\nabla((r, \alpha), (r', \alpha')) \sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right)\right).$$

It is easy to see that $\sum_{i=1}^{n}(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})) = 1$.

So,

$$I2TCA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$= ((r, \alpha), (r', \alpha')).$$

□

THEOREM 4.3. *(Boundedness). Given n I2LA* $((r_i, \alpha_i), (r_i', \alpha_i'))$, $i = 1, 2, \ldots, n$, *then,*

$$((r_{\sigma(n)}, \alpha_{\sigma(n)}), (r'_{\sigma(n)}, \alpha'_{\sigma(n)}))$$
$$\leq I2TCA_\mu(((r_1, \alpha_1), (r_1', \alpha_1')), ((r_2, \alpha_2), (r_2', \alpha_2')), \ldots, ((r_n, \alpha_n), (r_n', \alpha_n')))$$
$$\leq ((r_{\sigma(1)}, \alpha_{\sigma(1)}), (r'_{\sigma(1)}, \alpha'_{\sigma(1)})).$$

Proof. As we know $((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)})) \leq ((r_{\sigma(i-1)}, \alpha_{\sigma(i-1)}), (r'_{\sigma(i-1)}, \alpha'_{\sigma(i-1)}))$ for $i = 2, 3, \ldots n$.

Then $\nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)})) \leq \nabla((r_{\sigma(i-1)}, \alpha_{\sigma(i-1)}), (r'_{\sigma(i-1)}, \alpha'_{\sigma(i-1)}))$ for $i = 2, 3, \ldots n$.

We also know that $0 \leq (\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})) \leq 1$ for $i = 1, 2, 3, \ldots n$.

So,

$$\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$
$$\leq\left(\mu(H_{\sigma(i-1)})-\mu(H_{\sigma((i-1)-1)})\right)\nabla((r_{\sigma(i-1)},\alpha_{\sigma(i-1)}),(r'_{\sigma(i-1)},\alpha'_{\sigma(i-1)}))$$

for $i=2,3,\ldots n$.
Implies that

$$\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(n)},\alpha_{\sigma(n)}),(r'_{\sigma(n)},\alpha'_{\sigma(n)}))$$
$$\leq\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$

and

$$\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$
$$\leq\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(1)},\alpha_{\sigma(1)}),(r'_{\sigma(1)},\alpha'_{\sigma(1)})).$$

Implies that

$$\nabla((r_{\sigma(n)},\alpha_{\sigma(n)}),(r'_{\sigma(n)},\alpha'_{\sigma(n)}))\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)$$
$$\leq\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$

and

$$\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$
$$\leq\nabla((r_{\sigma(1)},\alpha_{\sigma(1)}),(r'_{\sigma(1)},\alpha'_{\sigma(1)}))\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right).$$

As we know that $\sum_{i=1}^{n}(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)}))=1$.
We can write

$$\nabla((r_{\sigma(n)},\alpha_{\sigma(n)}),(r'_{\sigma(n)},\alpha'_{\sigma(n)}))$$

$$\leq \sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right) \nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)}))$$

and

$$\sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right) \nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)})) \leq \nabla((r_{\sigma(1)}, \alpha_{\sigma(1)}), (r'_{\sigma(1)}, \alpha'_{\sigma(1)}))$$

So,

$$\Delta(\nabla((r_{\sigma(n)}, \alpha_{\sigma(n)}), (r'_{\sigma(n)}, \alpha'_{\sigma(n)}))) \leq \Delta\left(\sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right) \nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)}))\right)$$

and

$$\Delta\left(\sum_{i=1}^{n} \left(\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)})\right) \nabla((r_{\sigma(i)}, \alpha_{\sigma(i)}), (r'_{\sigma(i)}, \alpha'_{\sigma(i)}))\right) \leq \Delta(\nabla((r_{\sigma(1)}, \alpha_{\sigma(1)}), (r'_{\sigma(1)}, \alpha'_{\sigma(1)}))).$$

Implies

$$((r_{\sigma(n)}, \alpha_{\sigma(n)}), (r'_{\sigma(n)}, \alpha'_{\sigma(n)})) \leq I2TCA_{\mu}(((r_1, \alpha_1), (r'_1, \alpha'_1)), ((r_2, \alpha_2), (r'_2, \alpha'_2)), \ldots, ((r_n, \alpha_n), (r'_n, \alpha'_n)))$$

and

$$I2TCA_{\mu}(((r_1, \alpha_1), (r'_1, \alpha'_1)), ((r_2, \alpha_2), (r'_2, \alpha'_2)), \ldots, ((r_n, \alpha_n), (r'_n, \alpha'_n))) \leq ((r_{\sigma(1)}, \alpha_{\sigma(1)}), (r'_{\sigma(1)}, \alpha'_{\sigma(1)})).$$

Thus,

$$((r_{\sigma(n)}, \alpha_{\sigma(n)}), (r'_{\sigma(n)}, \alpha'_{\sigma(n)})) \leq I2TCA_{\mu}(((r_1, \alpha_1), (r'_1, \alpha'_1)), ((r_2, \alpha_2), (r'_2, \alpha'_2)), \ldots, ((r_n, \alpha_n), (r'_n, \alpha'_n))) \leq ((r_{\sigma(1)}, \alpha_{\sigma(1)}), (r'_{\sigma(1)}, \alpha'_{\sigma(1)})).$$

□

THEOREM 4.4. *(Commutativity). If* $((\acute{r}_1,\acute{\alpha}_1),(\acute{r}'_1,\acute{\alpha}'_1)),((\acute{r}_2,\acute{\alpha}_2),(\acute{r}'_2,\acute{\alpha}'_2)),\ldots,((\acute{r}_n,\acute{\alpha}_n),(\acute{r}'_n,\acute{\alpha}'_n))$ *is a permutation of* $((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n))$, *then*

$$I2TCA_\mu\left(((r_1,\alpha_1),(r'_1,\alpha'_1)),((r_2,\alpha_2),(r'_2,\alpha'_2)),\ldots,((r_n,\alpha_n),(r'_n,\alpha'_n))\right)$$
$$=I2TCA_\mu\left(((\acute{r}_1,\acute{\alpha}_1),(\acute{r}'_1,\acute{\alpha}'_1)),((\acute{r}_2,\acute{\alpha}_2),(\acute{r}'_2,\acute{\alpha}'_2)),\ldots,((\acute{r}_n,\acute{\alpha}_n),(\acute{r}'_n,\acute{\alpha}'_n))\right).$$

Proof. By applying the definition of σ such that $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$ such that $((r_{\sigma(1)},\alpha_{\sigma(1)}),(r'_{\sigma(1)},\alpha'_{\sigma(1)}))\geq((r_{\sigma(2)},\alpha_{\sigma(2)}),(r'_{\sigma(2)},\alpha'_{\sigma(2)}))\geq\cdots\geq((r_{\sigma(n)},\alpha_{\sigma(n)}),(r'_{\sigma(n)},\alpha'_{\sigma(n)}))$ where $x_{\sigma(i)}$ is the attribute corresponding to $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$, $H_{\sigma(i)}=\{x_{\sigma(k)}|k\leq i\}$ for $i\geq 1$, $H_{\sigma(0)}=\phi$.

We can say that $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))=((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}'_{\sigma(i)},\acute{\alpha}'_{\sigma(i)}))$, $H_{\sigma(i)}=H'_{\sigma(i)}$ for all $i=1,2,\ldots,n$; and $H_{\sigma(0)}=H'_{\sigma(0)}=\phi$.

$\Longrightarrow \nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))=\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}'_{\sigma(i)},\acute{\alpha}'_{\sigma(i)}))$ for all $i=1,2,\ldots,n$.

$\Longrightarrow$

$$\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$
$$=\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}'_{\sigma(i)},\acute{\alpha}'_{\sigma(i)}))$$

for all $i=1,2,\ldots,n$

$\Longrightarrow$

$$\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))$$

$$=\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}'_{\sigma(i)},\acute{\alpha}'_{\sigma(i)}))$$

for all $i=1,2,\ldots,n$

$\Longrightarrow$

$$\Delta\left(\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r'_{\sigma(i)},\alpha'_{\sigma(i)}))\right)$$

$$=\Delta\left(\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}'_{\sigma(i)},\acute{\alpha}'_{\sigma(i)}))\right)$$

for all $i=1,2,\ldots,n$.

Thus,

$$I2TCA_{\mu}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)$$
$$=I2TCA_{\mu}\left(((\acute{r}_1,\acute{\alpha}_1),(\acute{r}_1',\acute{\alpha}_1')),((\acute{r}_2,\acute{\alpha}_2),(\acute{r}_2',\acute{\alpha}_2')),\ldots,((\acute{r}_n,\acute{\alpha}_n),(\acute{r}_n',\acute{\alpha}_n'))\right).$$

□

THEOREM 4.5. *(Monotonicity). If* $((r_j,\alpha_j),(r_j',\alpha_j'))\leq((\acute{r}_j,\acute{\alpha}_j),(\acute{r}_j',\acute{\alpha}_j'))$ *for* $j=1,2,\ldots,n$, *then*

$$I2TCA_{\mu}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)$$
$$\leq I2TCA_{\mu}\left(((\acute{r}_1,\acute{\alpha}_1),(\acute{r}_1',\acute{\alpha}_1')),((\acute{r}_2,\acute{\alpha}_2),(\acute{r}_2',\acute{\alpha}_2')),\ldots,((\acute{r}_n,\acute{\alpha}_n),(\acute{r}_n',\acute{\alpha}_n'))\right).$$

Proof. By applying the definition of σ such that $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$ such that $((r_{\sigma(1)},\alpha_{\sigma(1)}),(r_{\sigma(1)}',\alpha_{\sigma(1)}'))\geq((r_{\sigma(2)},\alpha_{\sigma(2)}),(r_{\sigma(2)}',\alpha_{\sigma(2)}'))\geq\cdots\geq((r_{\sigma(n)},\alpha_{\sigma(n)}),(r_{\sigma(n)}',\alpha_{\sigma(n)}'))$, where $x_{\sigma(i)}$ is the attribute corresponding to $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))$, $H_{\sigma(i)}=\{x_{\sigma(k)}|k\leq i\}$ for $i\geq 1$, $H_{\sigma(0)}=\phi$.

We can say that $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\leq((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}_{\sigma(i)}',\acute{\alpha}_{\sigma(i)}'))$, $H_{\sigma(i)}=H_{\sigma(i)}'$ for all $i=1,2,\ldots,n$; and $H_{\sigma(0)}=H_{\sigma(0)}'=\phi$.

$\Longrightarrow\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\leq\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}_{\sigma(i)}',\acute{\alpha}_{\sigma(i)}'))$ for all $i=1,2,\ldots,n$.

$\Longrightarrow$

$$\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))$$
$$\leq\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}_{\sigma(i)}',\acute{\alpha}_{\sigma(i)}'))$$

for all $i=1,2,\ldots,n$

$\Longrightarrow$

$$\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))$$
$$\leq\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}_{\sigma(i)}',\acute{\alpha}_{\sigma(i)}'))$$

for all $i=1,2,\ldots,n$

$\Longrightarrow$

$$\Delta\left(\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)$$
$$\leq\Delta\left(\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\nabla((\acute{r}_{\sigma(i)},\acute{\alpha}_{\sigma(i)}),(\acute{r}_{\sigma(i)}',\acute{\alpha}_{\sigma(i)}'))\right)$$

for all $i = 1, 2, \ldots, n$.
Thus,

$$I2TCA_{\mu}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)$$
$$\leq I2TCA_{\mu}\left(((\acute{r}_1,\acute{\alpha}_1),(\acute{r}_1',\acute{\alpha}_1')),((\acute{r}_2,\acute{\alpha}_2),(\acute{r}_2',\acute{\alpha}_2')),\ldots,((\acute{r}_n,\acute{\alpha}_n),(\acute{r}_n',\acute{\alpha}_n'))\right).$$

□

5. GENERALIZED INTUITIONISTIC 2-TUPLE CORRELATED AVERAGING OPERATOR

Generalized intuitionistic 2-tuple correlated averaging aggregation operators with correlative weights are proposed, and some of its properties are studied.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the set of the attributes, and $\mu(x_i)$ $(i = 1, 2, \ldots, n)$ be the weight of the elements $x_i \in X$, where μ is a λ-fuzzy measure.

Definition 5.1. *Let $((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))$ be I2LA, X be the set of attributes, μ on X and $\kappa > 0$, then the generalized intuitionistic 2-tuple correlated averaging (GI2TCA) operator is defined as follows:*

$$GI2TCA_{\mu,\kappa}((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))$$
$$= \Delta\left(\sum_{i=1}^{n}\left(\mu(H_{\sigma(i)})-\mu(H_{\sigma(i-1)})\right)\left(\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)^{1/\kappa}\right)^{\kappa},$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$ such that $((r_{\sigma(1)},\alpha_{\sigma(1)}),(r_{\sigma(1)}',\alpha_{\sigma(1)}')) \geq ((r_{\sigma(2)},\alpha_{\sigma(2)}),(r_{\sigma(2)}',\alpha_{\sigma(2)}')) \geq \cdots \geq ((r_{\sigma(n)},\alpha_{\sigma(n)}),(r_{\sigma(n)}',\alpha_{\sigma(n)}'))$, $x_{\sigma(i)}$ is the attribute corresponding to $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))$, $H_{\sigma(i)} = \{x_{\sigma(k)} | k \leq i\}$ for $i \geq 1$, $H_{\sigma(0)} = \phi$.

We now consider some special cases of the GI2TCA operator. Let $((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))$ $(i = 1, 2, \ldots, n)$ be I2LA and μ be a fuzzy measure on X.

Case I. If $\mu(H) = 1$, for any $H \in P(X)$, then

$$GI2TCA_{\mu,\kappa}((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))$$
$$= ((r_{\sigma(1)},\alpha_{\sigma(1)}),(r_{\sigma(1)}',\alpha_{\sigma(1)}')).$$

Case II. If $\mu(H) = 0$, for any $H \in P(X)$ and $H \neq X$, then

$$GI2TCA_{\mu,\kappa}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)$$
$$= ((r_{\sigma(n)},\alpha_{\sigma(n)}),(r_{\sigma(n)}',\alpha_{\sigma(n)}')).$$

Case III. For any $A, B \in P(X)$ such that $|A| = |B|$, if $\mu(A) = \mu(B)$ and $\mu(H_{\sigma(i)}) = \frac{i}{n}$,

$1 \le i \le n$, then

$$
\begin{aligned}
&GI2TCA_{\mu,\kappa}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)\\
&\quad= \Delta\left(\sum_{i=1}^{n}\frac{1}{n}\left(\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)^{1/\kappa}\right)^{\kappa}\\
&\quad= \Delta\left(\sum_{i=1}^{n}\frac{1}{n}\left(\nabla((r_i,\alpha_i),(r_i',\alpha_i'))\right)^{1/\kappa}\right)^{\kappa}.
\end{aligned}
$$

Case IV. If conditions (2.4) and (4.1) hold, then the GI2TCA operator reduces to the following generalized intuitionistic 2-tuple weighted averaging (GI2TWA) operator:

$$
\begin{aligned}
&GI2TWA_{\mu,\kappa}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)\\
&\quad= \Delta\left(\sum_{i=1}^{n}\mu(x_i)\left(\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)^{1/\kappa}\right)^{\kappa}\\
&\quad= \Delta\left(\sum_{i=1}^{n}\mu(x_i)\left(\nabla((r_i,\alpha_i),(r_i',\alpha_i'))\right)^{1/\kappa}\right)^{\kappa}.
\end{aligned}
$$

In particular, if $\mu(x_i) = \frac{1}{n}$, for all $i = 1, 2, \ldots, n$, then the GI2TWA operator reduces to the generalized intuitionistic 2-tuple averaging (GI2TA) operator:

$$
\begin{aligned}
&GI2TA_{\mu,\kappa}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)\\
&\quad= \Delta\left(\sum_{i=1}^{n}\frac{1}{n}\left(\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)^{1/\kappa}\right)^{\kappa}\\
&\quad= \Delta\left(\sum_{i=1}^{n}\frac{1}{n}\left(\nabla((r_i,\alpha_i),(r_i',\alpha_i'))\right)^{1/\kappa}\right)^{\kappa}.
\end{aligned}
$$

Case V. If (4.3) and (4.2) hold, then the GI2TCA operator reduces to the generalized intuitionistic 2-tuple ordered weighted averaging (GI2TOWA) operator:

$$
\begin{aligned}
&GI2TOWA_{\mu,\kappa}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)\\
&\quad= \Delta\left(\sum_{i=1}^{n}w_i^{1/\kappa}\left(\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)\right)^{\kappa}.
\end{aligned}
$$

Case VI. If (4.5) and (4.4) hold, then the GI2TCA operator reduces to the following generalized intuitionistic 2-tuple weighted ordered weighted averaging (GI2TWOWA) operator:

$$
\begin{aligned}
&GITWOWA_{\mu,\kappa}\left(((r_1,\alpha_1),(r_1',\alpha_1')),((r_2,\alpha_2),(r_2',\alpha_2')),\ldots,((r_n,\alpha_n),(r_n',\alpha_n'))\right)\\
&\quad= \Delta\left(\sum_{i=1}^{n}w_i^{1/\kappa}\left(\nabla((r_{\sigma(i)},\alpha_{\sigma(i)}),(r_{\sigma(i)}',\alpha_{\sigma(i)}'))\right)\right)^{\kappa}.
\end{aligned}
$$

If $\mu(x_i) = \frac{1}{n}$ for $i = 1, 2, \ldots, n$, then the GI2TWOWA reduces to the GI2TOWA operator.
If $\kappa = 1$, then the GI2TCA operator reduces to the I2TCA operator.

REMARK 5.2. *Idempotent, boundedness, commutativity, and monotonicity hold for GI2TCA operator.*

6. APPLICATION IN MULTIPLE ATTRIBUTE GROUP DECISION MAKING

The I2TCA and the GI2TCA operators are applied to MAGDM problems based on the I2LI model. We also proposed the MAGDM method with intuitionistic linguistic arguments.

Let $D = \{D_1, D_2, \ldots, D_t\}$ be the set of DMs and $v = (v_1, v_2, \ldots, v_t)$ be weight vector of DMs, where $v_l \geq 0$, $l = 1, 2, \ldots, t$, $\sum_{l=1}^{t} v_l = 1$. Let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of alternatives, and $B = \{B_1, B_2, \ldots, B_n\}$ be the set of attributes.

Step 1. The DM D_l evaluates the alternative A_i with respect to the attribute B_j to get $r_{ij}^{(l)}$, then the decision matrices $\tilde{R}^{(l)} = [\tilde{r}_{ij}^{(l)}]_{m\times n}$ $(l = 1, 2, \ldots, t)$ are formed, where $\tilde{r}_{ij}^{(l)} = (r_{ij}^{(l)}, r_{ij}^{\prime(l)})$ is an intuitionistic linguistic element.

Step 2. Transform the intuitionistic linguistic decision matrix $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{m\times n}$ into the intuitionistic 2-tuple linguistic decision matrix

$$R^{(l)} = \left[\left(\left(r_{ij}^{(l)}, \alpha_{ij}^{(l)}\right), \left(r_{ij}^{\prime(l)}, \alpha_{ij}^{\prime(l)}\right)\right)\right]_{m\times n}.$$

Use the I2TWA operator to aggregate all the t DMs evaluation values $((r_{ij}^{(l)}, \alpha_{ij}^{(l)}), (r_{ij}^{\prime(l)}, \alpha_{ij}^{\prime(l)}))$, $l = 1, 2, \ldots, t$, to get the collective evaluation values $((r_{ij}, \alpha_{ij}), (r'_{ij}, \alpha'_{ij}))$ $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ as follows:

$$\begin{aligned}
&\left((r_{ij}, \alpha_{ij}), (r'_{ij}, \alpha'_{ij})\right) \\
&= I2TWA_v \left(\begin{array}{l} \left(\left(r_{ij}^{(1)}, \alpha_{ij}^{(1)}\right), \left(r_{ij}^{\prime(1)}, \alpha_{ij}^{\prime(1)}\right)\right), \left(\left(r_{ij}^{(2)}, \alpha_{ij}^{(2)}\right), \right. \\ \left. \left(r_{ij}^{\prime(2)}, \alpha_{ij}^{\prime(2)}\right)\right), \ldots, \left(\left(r_{ij}^{(t)}, \alpha_{ij}^{(t)}\right), \left(r_{ij}^{\prime(t)}, \alpha_{ij}^{\prime(t)}\right)\right) \end{array} \right) \\
&= \Delta \left(\sum_{l=1}^{n} v_l \nabla \left(\left(r_{ij}^{(l)}, \alpha_{ij}^{(l)}\right), \left(r_{ij}^{\prime(l)}, \alpha_{ij}^{\prime(l)}\right)\right) \right).
\end{aligned}$$

Step 3. Confirm the fuzzy measures of attributes of B. According to Equation 2.3, determine parameter λ of attributes. Fuzzy measures of remaining attributes of sets of B can be calculated by Equation 2.2. We use the I2TCA or GI2TCA operator to aggregate evaluation values to derive the overall values $((r_i, \alpha_i), (r'_i, \alpha'_i))$ $(i = 1, 2, \ldots, m)$ of the alternative A_i $(i = 1, 2, \ldots, m)$, concretely

$$\begin{aligned}&\left((r_i,\alpha_i),\left(r'_i,\alpha'_i\right)\right)\\&\quad= I2TCA_\mu\left(\left((r_{i1},\alpha_{i1}),\left(r'_{i1},\alpha'_{i1}\right)\right),\left((r_{i2},\alpha_{i2}),\left(r'_{i2},\alpha'_{i2}\right)\right),\ldots,\right.\\&\qquad\left.\left((r_{in},\alpha_{in}),\left(r'_{in},\alpha'_{in}\right)\right)\right)\\&\quad=\Delta\left(\sum_{j=1}^{n} w_{ij}\nabla\left(\left(\left(r_{i\sigma(j)},\alpha_{i\sigma(j)}\right),\left(r'_{i\sigma(j)},\alpha'_{i\sigma(j)}\right)\right)\right)\right),\end{aligned}$$

or

$$\begin{aligned}&\left((r_i,\alpha_i),\left(r'_i,\alpha'_i\right)\right)\\&\quad= GI2TCA_{\mu,\kappa}\left(\left((r_{i1},\alpha_{i1}),\left(r'_{i1},\alpha'_{i1}\right)\right),\left((r_{i2},\alpha_{i2}),\left(r'_{i2},\alpha'_{i2}\right)\right),\ldots,\right.\\&\qquad\left.\left((r_{in},\alpha_{in}),\left(r'_{in},\alpha'_{in}\right)\right)\right)\\&\quad=\Delta\left(\sum_{j=1}^{n} w_{ij}\left(\nabla\left(\left(\left(r_{i\sigma(j)},\alpha_{i\sigma(j)}\right),\left(r'_{i\sigma(j)},\alpha'_{i\sigma(j)}\right)\right)\right)\right)^{1/\kappa}\right)^{\kappa},\end{aligned}$$

where $((r_{i\sigma(1)},\alpha_{i\sigma(1)}),(r'_{i\sigma(1)},\alpha'_{i\sigma(1)})) \geq ((r_{i\sigma(2)},\alpha_{i\sigma(2)}),(r'_{i\sigma(2)},\alpha'_{i\sigma(2)})) \geq \cdots \geq ((r_{i\sigma(n)},\alpha_{i\sigma(n)}),(r'_{i\sigma(n)},\alpha'_{i\sigma(n)}))$ and $w_{ij}=\mu(H_{i\sigma(j)})-\mu(H_{i\sigma(j-1)})$. $H_{i\sigma(j)}$ is the set of j attributes corresponding to the $((r_{i\sigma(1)},\alpha_{i\sigma(1)}),(r'_{i\sigma(1)},\alpha'_{i\sigma(1)}))$, $((r_{i\sigma(2)},\alpha_{i\sigma(2)}),(r'_{i\sigma(2)},\alpha'_{i\sigma(2)})),\ldots,((r_{i\sigma(n)},\alpha_{i\sigma(n)}),(r'_{i\sigma(n)},\alpha'_{i\sigma(n)}))$.

Step 4. Greater the value $((r_i,\alpha_i),(r'_i,\alpha'_i))$ $(i=1,2,\ldots,m)$, better the alternative A_i.

7. EXAMPLE

Assume that a family wants to buy a house in best locality. There are five possible alternative locations where to buy: A_1 is Cavalry Ground; A_2 is Defence Housing Authority; A_3 is Johar Town; A_4 is The Model Town; A_5 is Gulberg.

Suppose that there are three family members/DMs (husband, wife, and daughter) D_i $(i=1,2,3)$, whose weight vector is $\upsilon=(0.2,0.5,0.3)$. Four attributes B_i $(i=1,2,3,4)$ are used to evaluate the alternatives: B_1, future appreciation in price of house; B_2, how safe is locality; B_3, social and political life in that locality, medical facility, roads, utilities, club, etc.; B_4, environmentally clean.

Step 1. The DMs evaluate the alternatives $(A_1, A_2, A_3, A_4, A_5)$ with respect to the attributes (B_1, B_2, B_3, B_4) in intuitionistic linguistic arguments to form decision matrices $(\tilde{R}^{(1)}, \tilde{R}^{(2)}, \tilde{R}^{(3)})$, as shown in Tables I–III, and $\kappa=3$.

Step 2. Transform each intuitionistic linguistic decision matrix $\tilde{R}^{(l)}=[\tilde{r}^{(l)}_{ij}]_{5\times 4}$ into the intuitionistic 2-tuple linguistic decision matrix

$$R^{(l)}=\left[\left(\left(r^{(l)}_{ij},0\right),\left(r'^{(l)}_{ij},0\right)\right)\right]_{5\times 4},$$

given in Tables IV–VII.

Table I. The decision matrix $\tilde{R}^{(1)}$

	B_1	B_2	B_3	B_4
A_1	(M,P)	(G,P)	(P,M)	(P,M)
A_2	(P,G)	(M,P)	(M,VP)	(G,VP)
A_3	(G,EP)	(M,VP)	(VG,EP)	(P,M)
A_4	(VG,VP)	(P,M)	(P,G)	(M,EP)
A_5	(EG,EP)	(P,G)	(VP,M)	(G,P)

Table II. The decision matrix $\tilde{R}^{(2)}$

	B_1	B_2	B_3	B_4
A_1	(P,M)	(VG,EP)	(VP,M)	(M,P)
A_2	(VP,M)	(P,P)	(G,VP)	(VG,EP)
A_3	(M,VP)	(P,M)	(G,EP)	(VP,M)
A_4	(EG,EP)	(M,VP)	(P,M)	(G,EP)
A_5	(G,VP)	(M,P)	(P,G)	(VG,EP)

Table III. The decision matrix $\tilde{R}^{(3)}$

	B_1	B_2	B_3	B_4
A_1	(G,EP)	(VG,EP)	(M,M)	(VP,M)
A_2	(M,P)	(P,P)	(VG,EP)	(M,P)
A_3	(P,M)	(VG,VP)	(G,EP)	(VP,G)
A_4	(G,VP)	(G,EP)	(P,M)	(M,VP)
A_5	(M,VP)	(P,M)	(M,VP)	(EG,EP)

Table IV. The decision matrix $R^{(1)}$

	B_1	B_2	B_3	B_4
A_1	((M,0),(P,0))	((G,0),(P,0))	((P,0),(M,0))	((P,0),(M,0))
A_2	((P,0),(G,0))	((M,0),(P,0))	((M,0),(VP,0))	((G,0),(VP,0))
A_3	((G,0),(EP,0))	((M,0),(VP,0))	((VG,0),(EP,0))	((P,0),(M,0))
A_4	((VG,0),(VP,0))	((P,0),(M,0))	((P,0),(G,0))	((M,0),(EP,0))
A_5	((EG,0),(EP,0))	((P,0),(G,0))	((VP,0),(M,0))	((G,0),(P,0))

Table V. The decision matrix $R^{(2)}$

	B_1	B_2	B_3	B_4
A_1	((P,0),(M,0))	((VG,0),(EP,0))	((VP,0),(M,0))	((M,0),(P,0))
A_2	((VP,0),(M,0))	((P,0),(P,0))	((G,0),(VP,0))	((VG,0),(EP,0))
A_3	((M,0),(VP,0))	((P,0),(M,0))	((G,0),(EP,0))	((VP,0),(M,0))
A_4	((EG,0),(EP,0))	((M,0),(VP,0))	((P,0),(M,0))	((G,0),(EP,0))
A_5	((G,0),(VP,0))	((M,0),(P,0))	((P,0),(G,0))	((VG,0),(EP,0))

Table VI. The decision matrix $\tilde{R}$

	B_1	B_2	B_3	B_4
A_1	((M,-0.2),(P,-0.1))	((VG,-0.2),(EP,0.4))	((P,-0.2),(M,0))	((P,0.2),(M,-0.5))
A_2	((P,-0.2),(M,-0.1)	((P,0.2),(P,0))	((G,0.1),(VP,-0.3))	((G,0.2),(VP,-0.2))
A_3	((M,-0.1),(VP,0.4)	((M,0.1),(P,0))	((G,0.2),(EP,0))	((VP,0.2),(M,0.3))
A_4	((VG,0.2),(VP,-0.5))	((M,0.1),(VP,0.1))	((P,0),(M,0.2))	((G,-0.5),(EP,0.3))
A_5	((G,0.1),(VP,-0.2))	((M,-0.5),(M,-0.3))	((P,0.1),(M,-0.1))	((VG,0.1),(EP,0.4))

Table VII. The decision matrix $R^{(3)}$

	B_1	B_2	B_3	B_4
A_1	((G,0),(EP,0))	((VG,0),(EP,0))	((M,0),(M,0))	((VP,0),(M,0))
A_2	((M,0),(P,0))	((P,0),(P,0))	((VG,0),(EP,0))	((M,0),(P,0))
A_3	((P,0),(M,0))	((VG,0),(VP,0))	((G,0),(EP,0))	((VP,0),(G,0))
A_4	((G,0),(VP,0))	((G,0),(EP,0))	((P,0),(M,0))	((M,0),(VP,0))
A_5	((M,0),(VP,0))	((P,0),(M,0))	((M,0),(VP,0))	((EG,0),(EP,0))

Use the I2TWA operator

$$\left((r_{ij}, \alpha_{ij}), (r'_{ij}, \alpha'_{ij})\right) = ITWA_v \begin{pmatrix} \left(\left(r_{ij}^{(1)}, \alpha_{ij}^{(1)}\right), \left(r_{ij}^{\prime(1)}, \alpha_{ij}^{\prime(1)}\right)\right), \left(\left(r_{ij}^{(2)}, \alpha_{ij}^{(2)}\right), \left(r_{ij}^{\prime(2)}, \alpha_{ij}^{\prime(2)}\right)\right), \\ \left(\left(r_{ij}^{(3)}, \alpha_{ij}^{(3)}\right), \left(r_{ij}^{\prime(3)}, \alpha_{ij}^{\prime(3)}\right)\right) \end{pmatrix},$$

$i = 1, 2, \ldots, 5,\ j = 1, 2, 3, 4.$ to aggregate all the decision matrices $\tilde{R}^{(i)}$ $(i = 1, 2, 3)$, into the collective decision matrix $\tilde{R}_{5\times4}$ as in Table VI.

Step 3. We determine fuzzy measures of attributes B_1, B_2, B_3, and B_4 and its λ parameter. Suppose that $\mu(B_1) = 0.3$, $\mu(B_2) = 0.25$, $\mu(B_3) = 0.37$, $\mu(B_4) = 0.2$. Then $\lambda = -0.2726$ by using Equation 2.3. According to Equation 2.2 fuzzy measures of attributes sets of $B = \{B_1, B_2, B_3, B_4\}$, we have $\mu(B_1, B_2) = 0.5295$, $\mu(B_1, B_3) = 0.6397$, $\mu(B_1, B_4) = 0.4836$, $\mu(B_2, B_3) = 0.5947$, $\mu(B_2, B_4) = 0.4363$, $\mu(B_3, B_4) = 0.5498$, $\mu(B_1, B_2, B_3) = 0.846$, $\mu(B_1, B_2, B_4) = 0.7006$, $\mu(B_1, B_3, B_4) = 0.8048$, $\mu(B_2, B_3, B_4) = 0.76235$, and $\mu(B_1, B_2, B_3, B_4) = 1$.

Then using the I2TCA operator

$$((r_i, \alpha_i), (r'_i, \alpha'_i)) = I2TCA_\mu((r_{i1}, \alpha_{i1}), (r'_{i1}, \alpha'_{i1})), ((r_{i2}, \alpha_{i2}), (r'_{i2}, \alpha'_{i2})), , ((r_{in}, \alpha_{in}), (r'_{in}, \alpha'_{in})))$$

to aggregate $((r_{ij}, \alpha_{ij}), (r'_{ij}, \alpha'_{ij}))$ $(j = 1, 2, 3, 4)$, we get the collective evaluation value to each alternative $A_i (i = 1, 2, \ldots, 5)$, as follows:

$$((r_1, \alpha_1), (r'_1, \alpha'_1)) = ((M, -0.102), (P, -0.043));$$

$$((r_3, \alpha_3), (r'_3, \alpha'_3)) = ((M, 0.164), (VP, 0.309));$$

$$((r_2, \alpha_2), (r'_2, \alpha'_2)) = ((M, 0.170), (P, -0.481));$$

$$((r_5, \alpha_5), (r'_5, \alpha'_5)) = ((M, 0.354), (P, -0.239));$$

$$((r_4, \alpha_4), (r'_4, \alpha'_4)) = ((M, 0.474), (VP, 0.402));$$

Since

$$\begin{aligned}((r_{i\sigma(1)}, \alpha_{i\sigma(1)}), (r'_{i\sigma(1)}, \alpha'_{i\sigma(1)}))\\ &= ((VG, -0.2), (EP, 0.4)), ((r_{i\sigma(2)}, \alpha_{i\sigma(2)}), (r'_{i\sigma(2)}, \alpha'_{i\sigma(2)}))\\ &= ((M, -0.2), (P, -0.1)), ((r_{i\sigma(3)}, \alpha_{i\sigma(3)}), (r'_{i\sigma(3)}, \alpha'_{i\sigma(3)}))\\ &= ((P, 0.2), (M, -0.5)), ((r_{i\sigma(4)}, \alpha_{i\sigma(4)}), (r'_{i\sigma(4)}, \alpha'_{i\sigma(4)}))\\ &= ((P, -0.2), (M, 0)),\end{aligned}$$

then

$$\begin{aligned}H_{i\sigma(1)} = \{B_2\}, H_{i\sigma(2)} = \{B_2, B_1\}, H_{i\sigma(3)}\\ = \{B_2, B_1, B_4\}, H_{i\sigma(4)} = \{B_2, B_1, B_4, B_3\}.\end{aligned}$$

We can get $w_{11} = 0.25$, $w_{12} = 0.2795$, $w_{13} = 0.171$, $w_{14} = 0.299$. Using the I2TCA operator, we get $((r_1, \alpha_1), (r'_1, \alpha'_1)) = ((\text{M}, -0.102), (\text{P}, -0.043))$. The rest of the collective evaluation values are the same as that for $((r_1, \alpha_1), (r'_1, \alpha'_1))$.
If the GI2TCA operator is used to calculate the collective values of the alternatives in step 3 and $\kappa = 3$, we can obtain

$$((r_1, \alpha_1), (r'_1, \alpha'_1)) = ((M, -0.207), (P, -0.177));$$

$$((r_3, \alpha_3), (r'_3, \alpha'_3)) = ((M, 0.070), (VP, 0.102));$$

$$((r_2, \alpha_2), (r'_2, \alpha'_2)) = ((M, 0.071), (VP, 0.414));$$

$$((r_5, \alpha_5), (r'_5, \alpha'_5)) = ((M, 0.249), (P, -0.397));$$

$$((r_4, \alpha_4), (r'_4, \alpha'_4)) = ((M, 0.358), (VP, 0.221)).$$

Step 4. Rank the alternatives $A_i (i = 1, 2, 3, 4, 5)$ with the largest $((r_i, \alpha_i), (r'_i, \alpha'_i))$ $(i = 1, 2, 3, 4, 5)$ to smallest according to the rules in Definition 3.7, we obtain

$$A_1 \prec A_3 \prec A_2 \prec A_5 \prec A_4.$$

Hence the best alternative is A_4, The Model Town.

8. CONCLUSIONS

We studied the situation where the attributes in the MADM problem are interactive or interdependent, and the evaluation values are I2LA. Aggregation operators using the Choquet integral are defined for I2LI, where the interdependence of attributes is considered. The properties of these new operators are studied, such as idempotency, commutativity, boundedness, and monotonicity. Afterward, the MAGDM method is defined where the 2-tuple linguistic information aggregated by these new operators. Finally, an illustrative example has been constructed to show the proposed MAGDM method. Our proposed method is different from all the previous techniques for group decision making due to the fact that the proposed method use intuitionistic 2-tuple fuzzy linguistic information, which will not cause any loss of information in the process. So it is efficient and feasible for real-world decision-making applications. In future, we shall continue working in the extension and application of the developed multicriteria group decision making with the Choquet integral and intuitionistic 2-tuple correlated geometric aggregation operator.

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