

# **Modified forms of some convex polytopes with constant metric dimension**



**By**

**Sadia Mehboob**

**DEPARTMENT OF MATHEMATICS**

**SCHOOL OF SCIENCES**

**UNIVERSITY OF MANAGEMENT & TECHNOLOGY**

**LAHORE, PAKISTAN**

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**By**

**Sadia Mehboob**

**Supervised by**

**Dr. Zohaib Zahid**

**Co-Supervised by**

**Dr. Tabasam Rashid**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

*In the name of Allah  
the most Gracious  
the most Compassionate*

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# Abstract

In this thesis, we expand the study of metric dimension by adding  $r$  times pendant edges and adding  $r$  times prisms to outer cycle of some convex polytopes determined in [21, 25]. These modified forms of convex polytopes have constant metric dimension and only three vertices appropriately chosen be suffice to resolve all the vertices of these graphs of convex polytopes.

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# Introduction

Graph theory is the famous branch of mathematics. It works as mathematical representations of several existing everyday problems. In 1736, Euler's used it at the first time for solution of the konigsberg bridge problem. Any physical situation can be represented by graphs. Also there are many branches of mathematics that have related with graph theory such as group theory, ring theory, numerical analysis, matrix theory, topology and probability. Graph is the combination of points and lines. It may be used to model a large variety of network structures. For example, in computer networks, servers and hosts can be denoted as vertices in a graph and edges can denoted as connections between them.

Metric dimension of graphs was first introduced by Slater [40] in 1975, and independently by Harary and Melter [15] in 1976, although, as indicated in [5], these concepts were studied and solved much earlier in 1963 by Erdős and Rényi [10] for